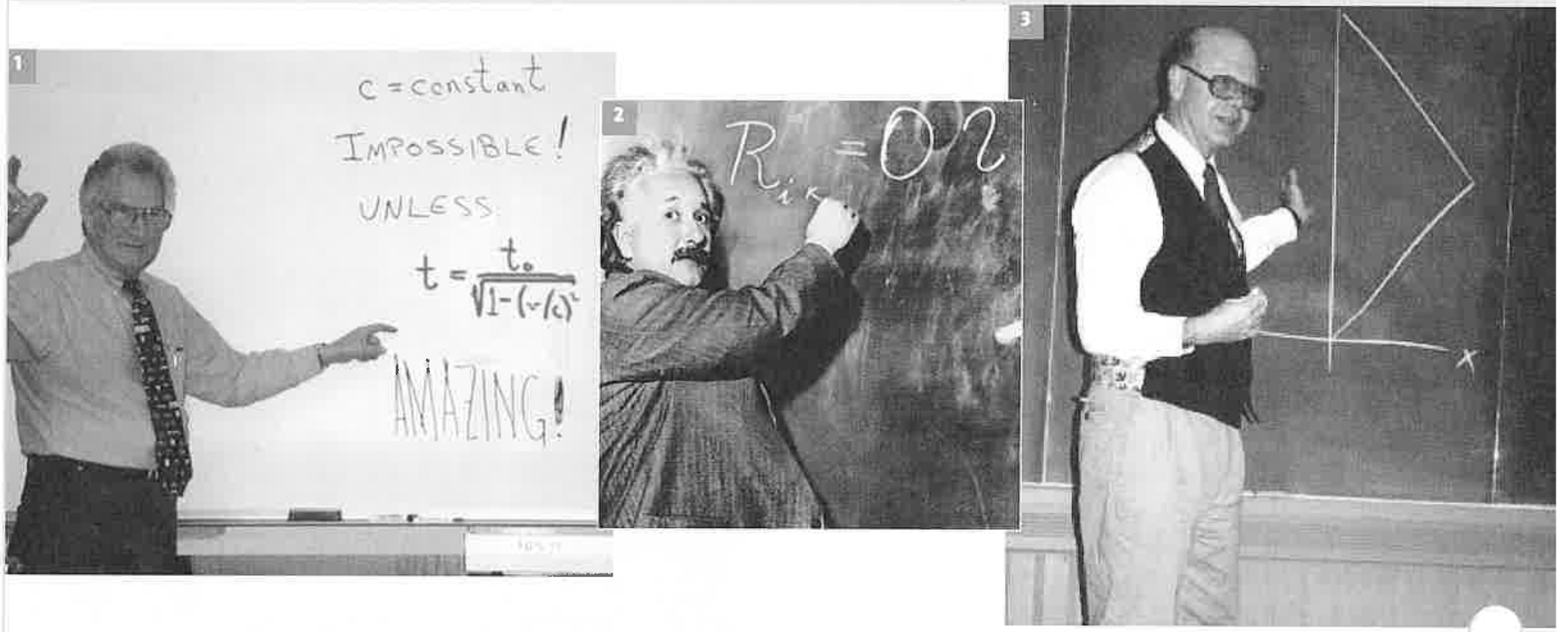
Part Eight
Relativity

Before the advent of Special Relativity, people thought the stars were beyond human reach. But distance is relative—it depends on motion. In a frame of reference moving almost as fast as light, distance contracts and time stretches enough to allow future astronauts access to the stars and beyond! We *are* like Evan's chickie on opening page 1, at the verge of a whole new beginning. Newton's physics got us to the moon; Einstein's physics points us to the stars. We live at an exciting time!

35 Special Theory of Relativity



1 Ken Ford, former CEO of the American Institute of Physics, brings the beauty of relativity to his high school students. 2 The twentieth-century's greatest scientist and one of its favorite human beings. 3 Edwin F. Taylor, co-author of several relativity books, gestures at a diagram showing two paths between a starting gun and crossing the finish line. One path is straight, one round-trip. The round-trip twin returns home younger than the lazy stay-at-home twin, an astonishing result showing Einstein's key idea that time between two events depends on the path taken between them.

Albert Einstein was born in Ulm, Germany, on March 14, 1879. According to popular legend, he was a slow child and learned to speak at a much later age than average; his parents feared for a while that he might be mentally retarded. Yet his elementary school records show that he was remarkably gifted in mathematics, physics, and playing the violin. He rebelled, however, at the practice of education by regimentation and rote and was expelled just as he was preparing to drop out at the age of 15. Largely because of business reasons, his family moved to Italy. Young Einstein renounced his German citizenship and went to live with family friends in Switzerland. There, two years younger than the normal age, he was allowed to take the entrance examinations for the renowned Swiss Federal Institute of Technology in Zurich. But because of difficulties with the French language, he did not pass the examination. He spent a year at a Swiss preparatory school in Aarau, where he was "promoted with protest in French." He tried the entrance exam again at Zurich and passed.

As an eager young student of physics in the 1890s, Albert Einstein was troubled by a difference between Newton's laws of mechanics and Maxwell's laws of electromagnetism. Newton's laws were independent of the state of motion of an observer; Maxwell's laws were not—or so it seemed. Someone at rest and someone in motion would find that the *same* laws of mechanics apply to a moving object being studied, but they would find that *different* laws of electricity and magnetism apply to a moving charge being studied. Newton's laws suggest that there is no such thing as absolute motion; only relative motion matters. But Maxwell's laws seemed to suggest that motion is absolute.

In a celebrated 1905 paper titled "On the Electrodynamics of Moving Bodies," written when he was 26, Einstein showed that Maxwell's laws can, after all, like Newton's laws, be interpreted as being independent of the state of motion of an observer—but at a cost! The cost of achieving this unified view of nature's laws is a total revolution in how we understand space and time.

Einstein showed that, as the forces between electric charges are affected by motion, the very measurements of space and time are also affected by motion. All measurements of space and time depend on relative motion. For example, the length of a rocket ship poised on its launching pad and the ticks of clocks within are found to change when the ship is set into motion at high speed. It has always been common sense that we change our position in space when we move, but Einstein flouted common sense and stated that, in moving, we also change our rate of proceeding into the future—time itself is altered. Einstein went on to show that a consequence of the interrelationship between space and time is an interrelationship between mass and energy, given by the famous equation $E = mc^2$.

These are the ideas that make up this chapter—the ideas of special relativity—ideas so remote from your everyday experience that understanding them requires stretching your mind. It will be enough to become acquainted with these ideas, so be patient with yourself if you don't understand them right away. Perhaps in some future era, when high-speed interstellar space travel is commonplace, your descendants will find that relativity makes common sense.

Motion Is Relative

Recall from Chapter 3 that whenever we talk about motion, we must always specify the vantage point from which motion is being observed and measured. For example, a person who walks along the aisle of a moving train may be walking at a speed of 1 kilometer per hour relative to his seat but at 60 kilometers per hour relative to the railroad station. We call the place from which motion is observed and measured a **frame of reference.** An object may have different velocities relative to different frames of reference.

To measure the speed of an object, we first choose a frame of reference and pretend that we are in that frame of reference standing still. Then we measure the speed with which the object moves relative to us—that is, relative to the frame of reference. In the foregoing example, if we measure from a position of rest within the train, the speed of the walking person is 1 kilometer per hour. If we measure from a position of rest on the ground, the speed of the walking person is 60 kilometers per hour. But the ground is not really still, for Earth spins like a top about its polar axis. Depending on how near the train is to the equator, the speed of the walking person may be as much as 1600 kilometers per hour relative to a frame of reference at the center of Earth. And the center of Earth is moving relative to the Sun. If we place our frame of reference on the Sun, the speed of the person walking in the train, which is on the orbiting Earth, is nearly 110,000 kilometers per hour. And the Sun is not at rest, for it orbits the center of our galaxy, which moves with respect to other galaxies.

MICHELSON–MORLEY EXPERIMENT

Isn't there some reference frame that is still? Isn't space itself still, and can't measurements be made relative to still space? In 1887, the American physicists A. A. Michelson and E. W. Morley attempted to answer these questions by performing an experiment that was designed to measure the motion of Earth through space. Because light travels in waves, it was then assumed that something in space vibrates—a mysterious something called *ether,* thought to fill all space and to serve as a frame of reference attached to space itself. These physicists used a very sensitive apparatus

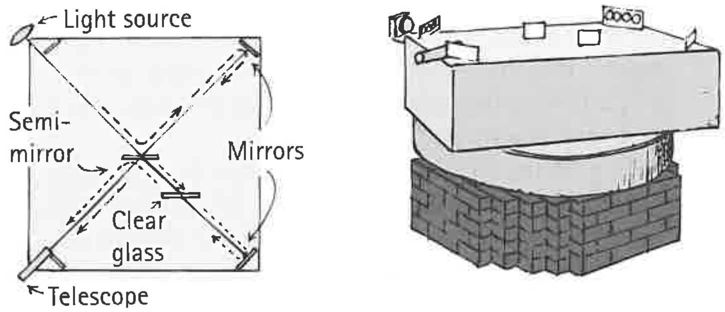


FIGURE 35.1

The Michelson–Morley interferometer, which splits a light beam into two parts and then recombines them to form an interference pattern after they have traveled different paths. Rotation was accomplished in their experiment by floating a massive sandstone slab in mercury. This schematic diagram shows how the half-silvered mirror splits the beam into two rays. The clear glass assured that both rays traverse the same amount of glass. In the actual experiment, four mirrors were placed at each corner to lengthen the paths.

called an *interferometer* to make their observations (Figure 35.1). In this instrument, a beam of light from a monochromatic source was separated into two beams with paths at right angles to each other; these were reflected and recombined to show whether there was any difference in average speed over the two back-and-forth paths. The interferometer was set with one path parallel to the motion of Earth in its orbit; then either Michelson or Morley carefully watched for any changes in average speed as the apparatus was rotated to put the other path parallel to the motion of Earth. The interferometer was sensitive enough to measure the difference in the round-trip times of light going with and against Earth's orbital velocity of 30 kilometers per second and going back and forth across Earth's path through space. But no changes were observed. None. Something was wrong with the sensible idea that the speed of light measured by a moving receiver should be its usual speed in a vacuum, c, plus or minus the contribution from the motion of the source or receiver. Many repetitions and variations of the Michelson–Morley experiment by many investigators showed the same null result. This was one of the puzzling facts of physics when the 20th century opened.

One interpretation of the bewildering result was suggested by the Irish physicist G. F. FitzGerald, who proposed that the length of the experimental apparatus shrank in the direction in which it was moving by just the amount required to counteract the presumed variation in the speed of light. The needed "shrinkage factor," $\sqrt{1 - v^2/c^2}$ was worked out by the Dutch physicist Hendrik A. Lorentz. This arithmetical factor accounted for the discrepancy, but neither FitzGerald nor Lorentz had a suitable theory for why this was so. Interestingly, the same factor was derived by Einstein in his 1905 paper, where he showed it to be the shrinkage factor of space itself, not just of matter in space.

How much the Michelson–Morley experiment influenced Einstein, if at all, is unclear. In any event, Einstein advanced the idea that the speed of light in free space is the same in all reference frames, an idea that was contrary to the classical ideas of space and time. Speed is a ratio of distance through space to a corresponding interval of time. For the speed of light to be a constant, the classical idea that space and time are independent of each other had to be rejected. Einstein saw that space and time are linked, and, with simple postulates, he developed a profound relationship between the two.

Postulates of the Special Theory of Relativity

Einstein saw no need for the ether. Gone with the stationary ether was the notion of an absolute frame of reference. All motion is relative, not to any stationary hitching post in the universe, but to arbitrary frames of reference. A rocket ship cannot measure its speed with respect to empty space but only with respect to other objects. If, for example, rocket ship A drifts past rocket ship B in empty space, spaceman A and spacewoman B will each observe the relative motion, and, from this observation, each will be unable to determine who is moving and who is at rest, if either.

This is a familiar experience to a passenger on a train who looks out his window and sees the train on the next track moving by his window. He is aware only of the relative motion between his train and the other train and cannot tell which train is moving. He may be at rest relative to the ground and the other train may be moving, or he may be moving relative to the ground and the other train may be at rest, or they both may be moving relative to the ground. The important point here is that, if you were in a train with no windows, there would be no way to determine whether the train was moving with uniform velocity or was at rest. This is the first of Einstein's postulates of the special theory of relativity:

All laws of nature are the same in all uniformly moving frames of reference.

On a jet airplane going 700 kilometers per hour, for example, coffee pours as it does when the plane is at rest; if we swing a pendulum in the moving plane, it swings as it would if the plane were at rest on the runway. There is no physical experiment that we can perform, even with light, to determine our state of uniform motion. The laws of physics within the uniformly moving cabin are the same as those in a stationary laboratory.

Any number of experiments can be devised to detect accelerated motion, but none can be devised, according to Einstein, to detect a state of uniform motion. Therefore, absolute motion has no meaning. It would be very peculiar if the laws of mechanics varied for observers moving at different speeds. It would mean, for example, that a pool player on a smoothly moving ocean liner would have to adjust her style of play to the speed of the ship, or even to the season as Earth varies in its orbital speed about the Sun. It is our common experience that no such adjustment is necessary. And, according to Einstein, this same insensitivity to motion extends to electromagnetism. No experiment, mechanical or electrical or optical, has ever revealed absolute motion. That is what the first postulate of relativity means.

One of the questions that Einstein, as a youth, asked himself was, "What would a light beam look like if you traveled along beside it?" According to classical physics, the beam would be at rest to such an observer. The more Einstein thought about this, the more convinced he became that one could not move with a light beam. He finally came to the conclusion that, no matter how fast two observers might be moving relative to each other, each of them would measure the speed of a light beam passing them to be 300,000 kilometers per second. This was the second postulate in his special theory of relativity:

The speed of light in free space has the same measured value for all observers, regardless of the motion of the source or the motion of the observer; that is, the speed of light is a constant.

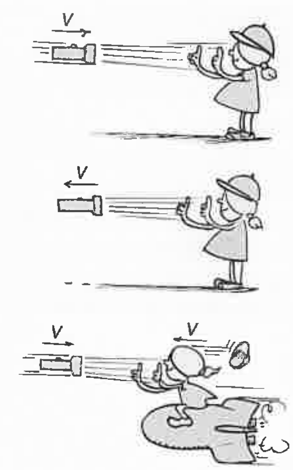


FIGURE 35.2
The speed of light is measured to be the same in all frames of reference.

To illustrate this statement, consider a rocket ship departing from the space station shown in Figure 35.3. A flash of light traveling at 300,000 km/s, or *c*, is emitted from the station. Regardless of the velocity of the rocket, an observer in the rocket sees the flash of light pass her at the same speed, *c*. If a flash is sent to the station from the moving rocket, observers on the station will measure the speed of the flash to be *c*. The speed of light is measured to be the same regardless of the speed of the source or receiver. *All* observers who measure the speed of light will find it has the same value, *c*. The more you think about this, the more you think it doesn't make sense. We will see that the explanation has to do with the relationship between space and time.

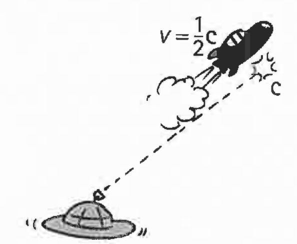


FIGURE 35.3
The speed of a light flash emitted by the space station is measured to be *c* by observers on both the space station and the rocket ship.

Simultaneity

An interesting consequence of Einstein's second postulate occurs with the concept of **simultaneity.** We say that two events are simultaneous if they occur at the same time. Consider, for example, a light source in the exact center of the

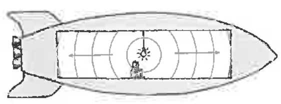


FIGURE 35.4

From the point of view of the observer who travels with the compartment, light from the source travels equal distances to both ends of the compartment and therefore strikes both ends simultaneously.

compartment of a rocket ship (Figure 35.4). When the light source is switched on, light spreads out in all directions at speed c. Because the light source is equidistant from the front and back ends of the compartment, an observer inside the compartment finds that light reaches the front end at the same instant it reaches the back end. This occurs whether the ship is at rest or moving at constant velocity. The events of hitting the back end and hitting the front end occur *simultaneously* for this observer within the rocket ship.

But what about an outside observer who views the same two events in another frame of reference—say, from a planet not moving with the ship? For that observer, these same two events are *not* simultaneous. As light travels out from the source, this observer sees the ship move forward, so the back of the compartment moves toward the beam while the front moves away from it. The beam going to the back of the compartment, therefore, has a shorter distance to travel than the beam going forward (Figure 35.5). Since the speed of light is the same in both directions, this outside observer sees the event of light hitting the back of the compartment *before* seeing the event of light hitting the front of the compartment. (Of course, we are making the assumption that the observer can discern these slight differences.) A little thought will show that an observer in another rocket ship that passes the ship in the opposite direction would report that the light reaches the front of the compartment first.

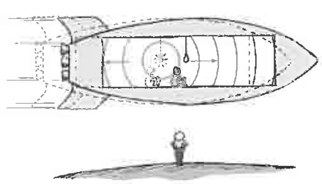


FIGURE 35.5

The events of light striking the front and back of the compartment are not simultaneous from the point of view of an observer in a different frame of reference. Because of the ship's motion, light that strikes the back of the compartment doesn't have as far to go and strikes sooner than light that strikes the front of the compartment.

Two events that are simultaneous in one frame of reference need not be simultaneous in a frame moving relative to the first frame.

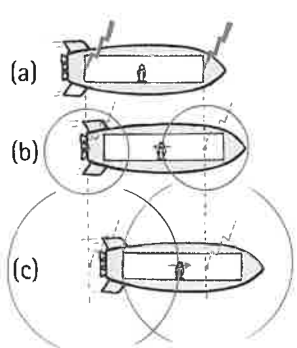
This nonsimultaneity of events in one frame that are simultaneous in another is a purely relativistic result—a consequence of light always having the same speed for all observers.

CHECK POINT

1. How is the nonsimultaneity of hearing thunder *after* seeing lightning similar to relativistic nonsimultaneity?
2. Suppose that the observer standing on a planet in Figure 35.5 sees a pair of lightning bolts simultaneously strike the front and rear ends of the compartment in the high-speed rocket ship. Will the lightning strikes be simultaneous to an observer in the middle of the compartment in the rocket ship? (We assume here that an observer can detect any slight differences in time for light to travel from the ends to the middle of the compartment.)

Check Your Answers

1. It isn't! The duration between hearing thunder and seeing lightning has nothing to do with moving observers or relativity. In such a case, you simply make corrections for the time the signals (sound and light) take to reach you. The relativity of simultaneity is a genuine discrepancy between observations made by observers in relative motion, and not simply a disparity between different travel times for different signals.
2. No; an observer in the middle of the compartment will see the lightning that hits the front end of the compartment before seeing the lightning that hits the rear end. This is shown in positions (a), (b), and (c) to the left. In (a), we see both lightning bolts striking the ends of the compartment simultaneously, according to the outside observer. In position (b), light from the front lightning bolt reaches the observer within the rocket ship. Slightly later, in (c), light from the rear lightning bolt reaches this observer.



Spacetime

When we look up at the stars, we realize that we are actually looking backward in time. The stars we see farthest away are the stars we are seeing longest ago. The more we think about this, the more apparent it becomes that space and time must be intimately tied together.

The space we live in is three-dimensional; that is, we can specify the position of any location in space with three dimensions. For example, these dimensions could be north–south, east–west, and up–down. If we are at the corner of a rectangular room and wish to specify the position of any point in the room, we can do this with three numbers. The first would be the number of meters the point is along a line joining the side wall and the floor; the second would be the number of meters the point is along a line joining the adjacent back wall and the floor; and the third would be the number of meters the point lies above the floor or along the vertical line joining the walls at the corner. Physicists speak of these three lines as the *coordinate axes* of a reference frame (Figure 35.6). Three numbers—the distances along the *x* axis, the *y* axis, and the *z* axis—will specify the position of a point in space.

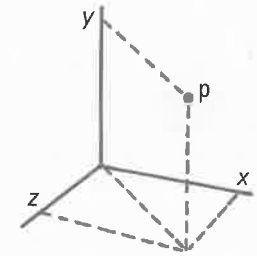


FIGURE 35.6
Point P can be specified with three numbers: the distances along the *x* axis, the *y* axis, and the *z* axis.

We also use three dimensions to specify the size of objects. A box, for example, is described by its length, width, and height. But the three dimensions do not give a complete picture. There is a fourth dimension—time. The box was not always a box of given length, width, and height. It began as a box only at a certain point in time, on the day it was made. Nor will it always be a box. At any moment, it may be crushed, burned, or otherwise destroyed. So the three dimensions of space are a valid description of the box only during a certain specified period of time. We cannot speak meaningfully about space without implying time. Things exist in **spacetime.** Each object, each person, each planet, each star, each galaxy exists in what physicists call "the spacetime continuum."

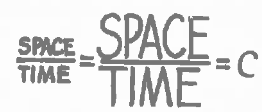


FIGURE 35.7
All space and time measurements of light are unified by *c*.

Two side-by-side observers at rest relative to each other share the same reference frame. Both would agree on measurements of space and time intervals between given events, so we say they share the same realm of spacetime. If there is relative motion between them, however, the observers will not agree on these measurements of space and time. At ordinary speeds, differences in their measurements are imperceptible, but, at speeds near the speed of light—so-called relativistic speeds—the differences are appreciable. Each observer is in a different realm of spacetime, and one observer's measurements of space and time differ from the measurements of another observer in some other realm of spacetime. The measurements differ not haphazardly but in such a way that each observer will always measure the same ratio of space and time for light: the greater the measured distance in space, the greater the measured interval of time. This constant ratio of space and time for light, *c*, is the unifying factor between different realms of spacetime and is the essence of Einstein's second postulate.

Time Dilation

Let's examine the notion that time can be stretched. Imagine that we are somehow able to observe a flash of light bouncing to and fro between a pair of parallel mirrors, like a ball bouncing to and fro between a floor and ceiling. If the distance between the mirrors is fixed, then the arrangement constitutes a *light clock,* because the back-and-forth trips of the flash take equal time intervals (Figure 35.8). Suppose this light clock is inside a transparent, high-speed spaceship. An observer who travels along with the ship and watches the light clock

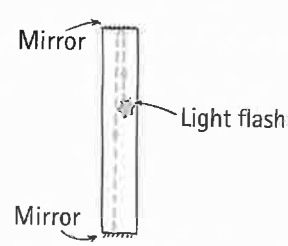


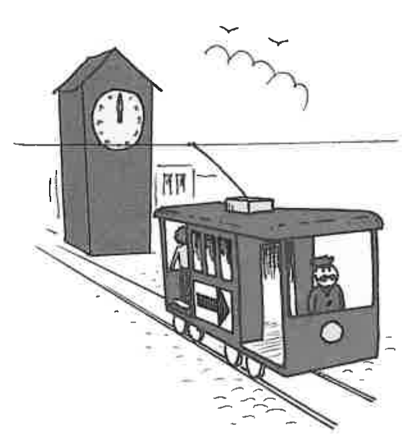
FIGURE 35.8
A light clock. A flash of light will bounce up and down between parallel mirrors and "tick off" equal intervals of time.

Clockwatching on a Trolley Car Ride

Pretend you are Einstein at the turn of the 20th century, riding in a trolley car moving away from a huge clock in the village square. The clock reads 12 noon. To say it reads 12 noon is to say that light that carries the information "12 noon" is reflected by the clock and travels toward you along your line of sight. If you suddenly move your head to the side, the light carrying the information, instead of meeting your eye, continues past, presumably out into space. Out there, an observer who *later* receives the light says, "Oh, it's 12 noon on Earth now." But, from your point of view, it's now later than that. You and the distant observer see 12 noon at different times. You wonder more about this idea. If the trolley car traveled as fast as the light, then the trolley car would keep up with the light's information that says "12 noon." Traveling at the speed of light, then, tells you it's always 12 noon at the village square. In other words, time at the village square is frozen!

If the trolley car is not moving, you see the village square clock move into the future at the rate of 60 s/min; if you move at the speed of light, you see seconds on the clock taking infinite time. These are the two extremes. What's in between? How would the advance of the clock's hands be viewed as you move at speeds less than the speed of light?

A little thought will show that you will receive the message "1 o'clock" anywhere from 60 minutes to an infinity of time after you receive the message "12 noon," depending on what your speed is between the extremes of zero and the speed of light. From your high-speed (but less than *c*) frame of reference, you see all events taking place in the reference frame of the clock (which is Earth) as happening in slow motion. If you reverse direction and travel at high speed back toward the clock, you'll see all events taking place in the clock's reference frame as being speeded up. When you return and are once again sitting in the square, will the effects of going and coming compensate each other? Amazingly, no! Time will be stretched. The wristwatch you were wearing the whole time and the village clock will disagree. This is time dilation.



(Figure 35.9a) sees the flash reflecting straight up and down between the two mirrors, just as it would if the spaceship were at rest. This observer sees no unusual effects. Note that because the observer is in the ship moving along with it, there is no relative motion between the observer and the light clock; we say that the observer and the clock share the same reference frame in spacetime.

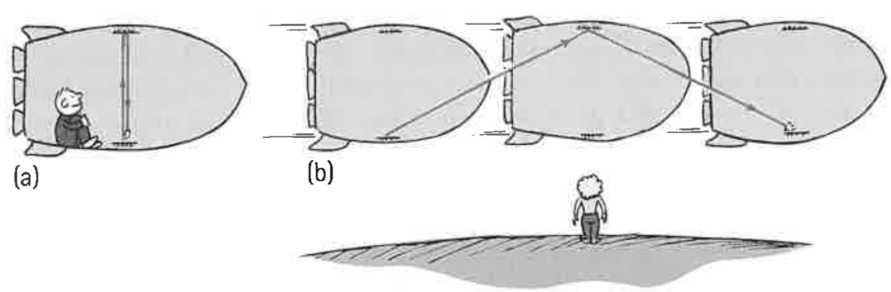


FIGURE 35.9
INTERACTIVE FIGURE
(a) An observer moving with the spaceship observes the light flash moving vertically between the mirrors of the light clock.
(b) An observer who sees the moving ship pass by observes the flash moving along a diagonal path.

Suppose now that we are standing on the ground as the spaceship whizzes by us at a high speed—say, half the speed of light. Things are quite different from our reference frame, for we do not see the light path as being simple up-and-down motion. Because each flash moves horizontally while it moves vertically between the two mirrors, we see the flash follow a diagonal path. Notice in Figure 35.9b that from our earthbound frame of reference, the flash travels a *longer distance* as it makes one round-trip between the mirrors, considerably longer than the distance it travels in the reference frame of the observer riding along with the ship. Because the speed of light is the same in all reference frames (Einstein's second postulate), the flash must travel for a correspondingly longer time between the mirrors in our frame than in the reference frame of the onboard observer. This follows from the definition of speed—distance divided by time. *The longer diagonal distance must be divided by a* *correspondingly longer time interval to yield an unvarying value for the speed of light.* This stretching out of time is called **time dilation.**

We have considered a light clock in our example, but the same is true for any kind of clock. All clocks run more slowly when moving than when at rest. Time dilation has to do not with the mechanics of clocks, but with the nature of time itself.

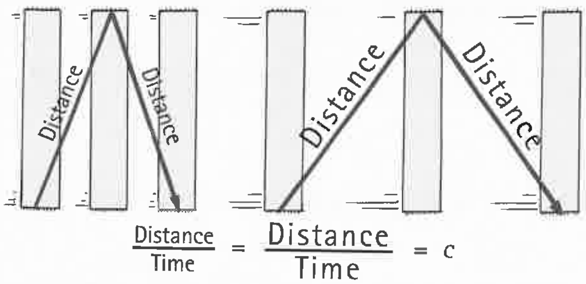


FIGURE 35.10
INTERACTIVE FIGURE
The longer distance covered by the light flash in following the longer diagonal path on the right must be divided by a correspondingly longer time interval to yield an unvarying value for the speed of light.

The relationship of time dilation for different frames of reference in spacetime can be derived from Figure 35.10 with simple geometry and algebra.[1] The relationship between the time t_0 (call it the *proper time*) in the frame of reference moving with the clock and the time t measured in another frame of reference (call it the *relative time*) is

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[1]The light clock is shown in three successive positions in the figure below. The diagonal lines represent the path of the light flash as it starts from the lower mirror at position 1, moves to the upper mirror at position 2, and then back to the lower mirror at position 3. Distances on the diagram are marked ct, vt, and ct_0, which follows from the fact that the distance traveled by a uniformly moving object is equal to its speed multiplied by the time.

The symbol t_0 represents the time it takes for the flash to move between the mirrors, as measured from a frame of reference fixed to the light clock. This is the time for straight up or down motion. The speed of light is c, and the path of light is seen to move a vertical distance ct_0. This distance between mirrors is at right angles to the motion of the light clock and is the same in both reference frames.

The symbol t represents the time it takes the flash to move from one mirror to the other, as measured from a frame of reference in which the light clock moves with speed v. Because the speed of the flash is c and the time it takes to go from position 1 to position 2 is t, the diagonal distance traveled is ct. During this time t, the clock (which travels horizontally at speed v) moves a horizontal distance vt from position 1 to position 2.

As the figure shows, these three distances make up a right triangle in which ct is the hypotenuse and ct_0 and vt are legs. A well-known theorem of geometry, the Pythagorean Theorem, states that the square of the hypotenuse is equal to the sum of the squares of the two legs. If we apply this formula to the figure, we obtain:

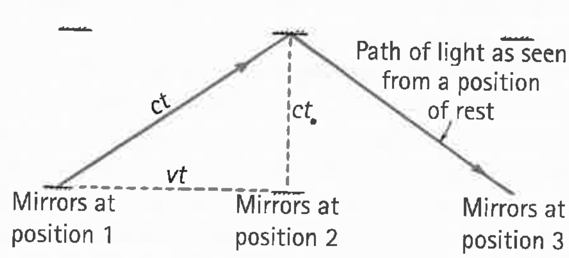
$$c^2t^2 = c^2t_0^2 + v^2t^2$$

$$c^2t^2 - v^2t^2 = c^2t_0^2$$

$$t^2[1 - (v^2/c^2)] = t_0^2$$

$$t^2 = \frac{t_0^2}{1 - (v^2/c^2)}$$

$$t = \frac{t_0}{\sqrt{1 - (v^2/c^2)}}$$



where v represents the speed of the clock relative to the outside observer (the same as the relative speed of the two observers) and c is the speed of light. The quantity

$$\sqrt{1 - \frac{v^2}{c^2}}$$

is the same factor used by Lorentz to explain length contraction. We call the inverse of this quantity the *Lorentz factor*, γ (gamma). That is,

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Then we can express the time dilation equation more simply as

$$t = \gamma t_0$$

Let's look at the terms in γ. Some mental tinkering will show that γ is always greater than 1 for any speed v greater than zero. Note that, since speed v is always less than c, the ratio v/c is always less than 1; likewise for v^2/c^2. Can you see it follows that γ is greater than 1? Now consider the case where $v = 0$. This ratio v^2/c^2 is 0, and for everyday speeds, where v is negligibly small compared to c, it's practically 0. Then $1 - v^2/c^2$ has a value of 1, as has $\sqrt{1 - v^2/c^2}$, which makes $\gamma = 1$. Then we find $t = t_0$—time intervals appear the same in both reference frames. For higher speeds, v/c is between 0 and 1, and $1 - v^2/c^2$ is less than 1; likewise, $\sqrt{1 - v^2/c^2}$. This makes γ greater than 1, so t_0 multiplied by a factor greater than 1 produces a value greater than t_0—an elongation—a dilation of time.

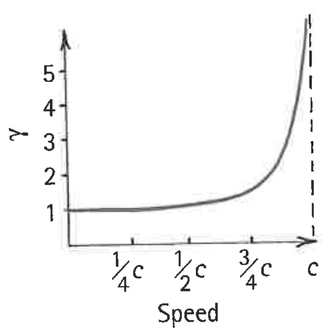


FIGURE 35.11

A plot of the Lorentz factor γ as a function of speed.

To consider some numerical values, assume that v is 50% the speed of light. Then we substitute $0.5c$ for v in the time-dilation equation and, after some arithmetic, find that $\gamma = 1.15$; so $t = 1.15t_0$. This means that if we viewed a clock on a spaceship traveling at half the speed of light, we would see the second hand take 1.15 minutes to make a revolution, whereas an observer riding with the clock would see it take 1 minute. If the spaceship passes us at 87% the speed of light, $\gamma = 2$ and $t = 2t_0$. We would measure time events on the spaceship taking twice the usual intervals, for the hands of a clock on the ship would turn only half as fast as those on our own clock. Events on the ship would seem to take place in slow motion. At 99.5% the speed of light, $\gamma = 10$ and $t = 10t_0$; we would see the second hand of the spaceship's clock take 10 minutes to sweep through a revolution requiring 1 minute on our clock.

To put these figures another way, at $0.995c$, the moving clock would appear to run a tenth of our rate; it would tick only 6 seconds while our clock ticks 60 seconds. At $0.87c$, the moving clock ticks at half rate and shows 30 seconds to our 60 seconds; at $0.50c$, the moving clock ticks 1/1.15 as fast and ticks 52 seconds to our 60 seconds. Moving clocks run slow.

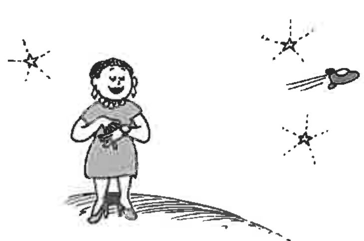


FIGURE 35.12

When we see the rocket at rest, we see it traveling at the maximum rate in time: 24 hours per day. When we see the rocket traveling at close to the maximum rate through space (the speed of light), we see its time practically standing still.

Nothing is unusual about a moving clock itself; it is simply ticking to the rhythm of a different time. The faster a clock moves, the slower it appears to run as viewed by an observer not moving with the clock. If it were possible to make a clock fly by us at the speed of light, the clock would not appear to be running at all. We would measure the interval between ticks to be infinite. The clock would be ageless! But one thing *does* move at the speed of light—light itself. So photons never age. There is no passage of time for a photon. Photons are truly ageless.

If a person whizzing past us were to check a clock in our reference frame, he would find our clock to be running as slowly as we would find his to be. Each would conclude that the other's clock runs slow. There is really no contradiction here, for it is physically impossible for two observers in relative motion to refer to one and the same realm of spacetime. The measurements made in one realm of spacetime need not agree with the measurements made in another realm of spacetime. The measurement that all observers always agree on, however, is the speed of light.

Time dilation has been confirmed in the laboratory innumerable times with particle accelerators. The lifetimes of fast-moving radioactive particles increase as the speed goes up, and the amount of increase is just what Einstein's equation predicts. Time dilation has been confirmed also for not-so-fast motion. In 1971, to test Einstein's theory, four cesium-beam atomic clocks were twice flown on regularly scheduled commercial jet flights around the world, once eastward and once westward, to test Einstein's theory of relativity with macroscopic clocks. The clocks indicated different times after their round-trips. Relative to the atomic time scale of the U.S. Naval Observatory, the observed time differences, in billionths of a second, were in accord with Einstein's prediction. Now, with atomic clocks orbiting Earth as part of the Global Positioning System, adjustments for the effects of time dilation are essential in order to use signals from the clocks to pinpoint locations on Earth.

This all seems very strange to us only because it is not our common experience to deal with measurements made at relativistic speeds or atomic-clock-type measurements at ordinary speeds. The theory of relativity does not make common sense. But common sense, according to Einstein, is that layer of prejudices laid down in the mind prior to the age of 18. If we had spent our youth zapping through the universe in high-speed spaceships, we would probably be quite comfortable with the results of relativity.

fyi

- The Global Positioning System (GPS) takes account of the time dilation of orbiting atomic clocks. Otherwise, your GPS receiver would badly miss your location.

CHECK POINT

1. If you were moving in a spaceship at a high speed relative to Earth, would you notice a difference in your pulse rate? In the pulse rate of the people back on Earth?
2. Will observers A and B agree on measurements of time if A moves at half the speed of light relative to B? If both A and B move together at half the speed of light relative to Earth?
3. Does time dilation mean that time really passes more slowly in moving systems or only that it seems to pass more slowly?

Check Your Answers

1. There would be no relative speed between you and your pulse because the two share the same frame of reference. Therefore, you would notice no relativistic effects in your pulse. There would be, however, a relativistic effect between you and people back on Earth. You would find their pulse rate to be slower than normal (and they would find your pulse rate to be slower than normal). Relativity effects are always attributed to the other guy.
2. When A and B move relative to each other, each observes a slowing of time in the other's frame of reference. So they do not agree on measurements of time. When they are moving in unison, they share the same frame of reference and agree on measurements of time. They see each other's time as passing normally, and they each see events on Earth in the same slow motion.
3. The slowing of time in moving systems is not merely an illusion resulting from motion. Time really does pass more slowly in a moving system relative to one at relative rest, as we shall see in the next section. Read on!

The Twin Trip

A dramatic illustration of time dilation is provided by identical twins, one an astronaut who takes a high-speed round-trip journey in the galaxy while the other stays home on Earth. When the traveling twin returns, he is younger than the stay-at-home twin. How much younger depends on the relative speeds involved.

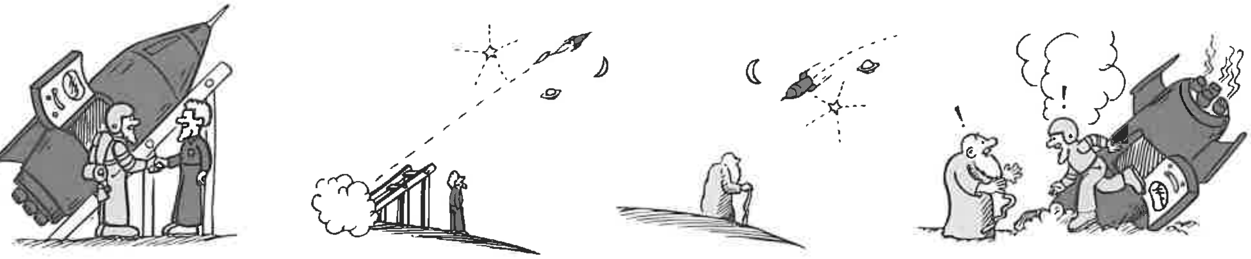


FIGURE 35.13
The traveling twin does not age as fast as the stay-at-home twin.

If the traveling twin maintains a speed of 50% the speed of light for 1 year (according to clocks aboard the spaceship), 1.15 years will have elapsed on Earth. If the traveling twin maintains a speed of 87% the speed of light for a year, then 2 years will have elapsed on Earth. At 99.5% the speed of light, 10 Earth years would pass in one spaceship year. At this speed, the traveling twin would age a single year while the stay-at-home twin would age 10 years.

fyi

- Cosmonaut Sergei Avdeyev spent more than two years orbiting Earth in the *Mir* spacecraft, and, due to time dilation, he is today two-hundredths of a second younger than he would be if he'd never been in space!

One question often arises: Since motion is relative, why doesn't the effect work equally well the other way around? Why wouldn't the traveling twin return to find his stay-at-home twin younger than himself? We will show that, from the frames of reference of both the earthbound twin and traveling twin, it is the earthbound twin who ages more. First, consider a spaceship hovering at rest relative to Earth. Suppose the spaceship sends brief, regularly spaced flashes of light to the planet (Figure 35.14). Some time will elapse before the flashes get to the planet, just as 8 minutes elapse before sunlight gets to Earth. The light flashes will encounter the receiver on the planet at speed c. Since there is no relative motion between the sender and receiver, successive flashes will be received as frequently as they are sent. For example, if a flash is sent from the ship every 6 minutes, then, after some initial delay, the receiver will receive a flash every 6 minutes. With no motion involved, there is nothing unusual about this.

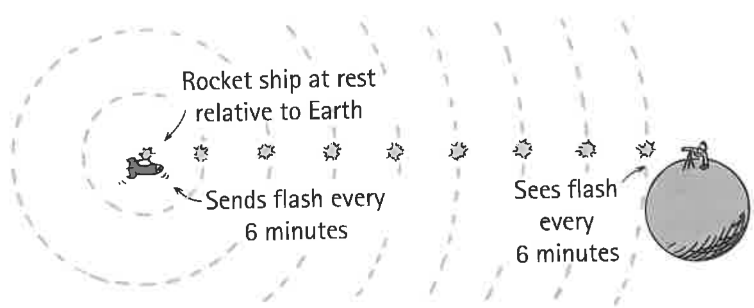


FIGURE 35.14
When no motion is involved, the light flashes are received as frequently as the spaceship sends them.

When motion is involved, the situation is quite different. It is important to note that the speed of the flashes will still be c, no matter how the ship or receiver may move. How frequently the flashes are seen, however, very much depends on the relative motion involved. When the ship travels toward the receiver, the receiver sees the flashes more frequently. This happens not only because time is altered due to motion but mainly because each succeeding flash has less distance to travel as the ship gets closer to the receiver. If the spaceship emits a flash every 6 minutes, the flashes will be seen at intervals of less than 6 minutes. Suppose the ship is traveling fast enough for the flashes to be seen twice as frequently. Then they are seen at intervals of 3 minutes (Figure 35.15).

If the ship recedes from the receiver at the same speed and still emits flashes at 6-minute intervals, these flashes will be seen half as frequently by the receiver—that is, at 12-minute intervals (Figure 35.16). This is mainly because each suc-

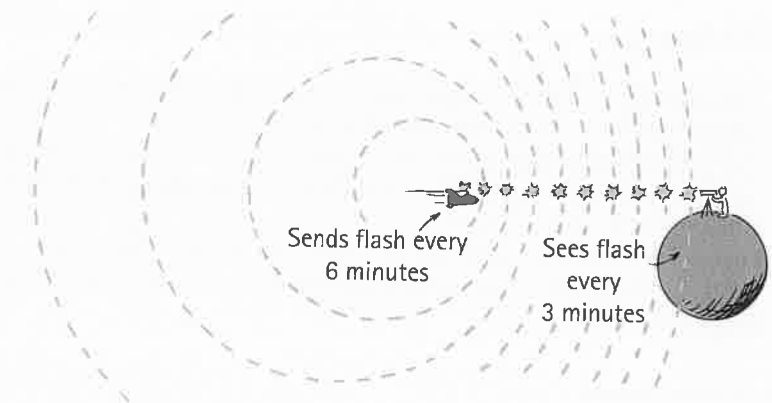


FIGURE 35.15

When the sender moves toward the receiver, the flashes are seen more frequently.

ceeding flash has a longer distance to travel as the ship gets farther away from the receiver.

The effect of moving away is just the opposite of moving closer to the receiver. So if the flashes are received twice as frequently when the spaceship is approaching (6-minute flash intervals are seen every 3 minutes), they are received half as frequently when it is receding (6-minute flash intervals are seen every 12 minutes).[2]

This means that, if two events are separated by 6 minutes according to the spaceship clock, flashes will be seen to be separated by 12 minutes when the spaceship recedes and by only 3 minutes when the ship is approaching.

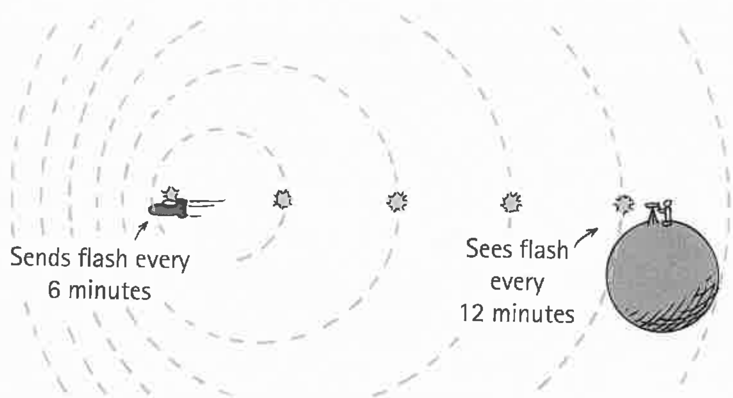


FIGURE 35.16

When the sender moves away from the receiver, the flashes are spaced farther apart and are seen less frequently.

[2]This reciprocal relationship (halving and doubling of frequencies) is a consequence of the constancy of the speed of light and can be illustrated with the following example: Suppose that a sender on Earth emits flashes 3 min apart to a distant observer on a planet that is at rest relative to Earth. The observer, then, sees a flash every 3 min. Now suppose a second observer travels in a spaceship between Earth and the planet at a speed great enough to allow him to see the flashes half as frequently—6 min apart. This halving of frequency occurs for a speed of recession of $0.6c$. We can see that the frequency will double for a speed of approach of $0.6c$ by supposing that the spaceship emits its own flash every time it sees an Earth flash—that is, every 6 min. How does the observer on the distant planet see these flashes? Since Earth flashes and the spaceship flashes travel together at the same speed, c, the observer will see not only Earth flashes every 3 min but the spaceship flashes every 3 min as well. So, although a person on the spaceship emits flashes every 6 min, the observer sees them every 3 min at twice the emitting frequency. So, for a speed of recession where frequency appears halved, frequency appears doubled for the same speed of approach. If the ship were traveling faster so that the frequency of recession were 1/3 or 1/4 as much, then the frequency of approach would be threefold or fourfold, respectively. This reciprocal relationship does not hold for waves that require a medium. In the case of sound waves, for example, a speed that results in a doubling of emitting frequency for approach produces 2/3 (not 1/2) the emitting frequency for recession. So the relativistic Doppler effect differs from the one we experience with sound.

CHECK POINT

1. If the spaceship emits an initial "starting gun" signal followed by a flash every 6 min for an hour, how many flashes will be emitted?
2. The ship sends equally spaced flashes every 6 min while approaching the receiver at constant speed. Will these flashes be equally spaced when they encounter the receiver?
3. If the receiver sees these flashes at 3-min intervals, how much time will elapse between the initial signal and the last flash (in the frame of reference of the receiver)?

Check Your Answers

1. The ship will emit a total of 10 flashes in 1 h, since (60 min)/(6 min) = 10 (11 if the initial signal is counted).
2. Yes; as long as the ship moves at constant speed, the equally spaced flashes will be seen equally spaced but more frequently. (If the ship accelerated while sending flashes, then they would not be seen at equally spaced intervals.)
3. Thirty minutes, since the 10 flashes are coming every 3 min.

Let's apply this doubling and halving of flash intervals to the twins. Suppose the traveling twin recedes from the earthbound twin at the same high speed for 1 hour and then quickly turns around and returns in 1 hour. Follow this line of reasoning with the help of Figure 35.17. The traveling twin takes a round trip of 2 hours, according to all clocks aboard the spaceship. This trip will not be seen to take 2 hours from the Earth frame of reference, however. We can see this with the help of the flashes from the ship's light clock.

As the ship recedes from Earth, it emits a flash of light every 6 minutes. These flashes are received on Earth every 12 minutes. During the hour of going away from

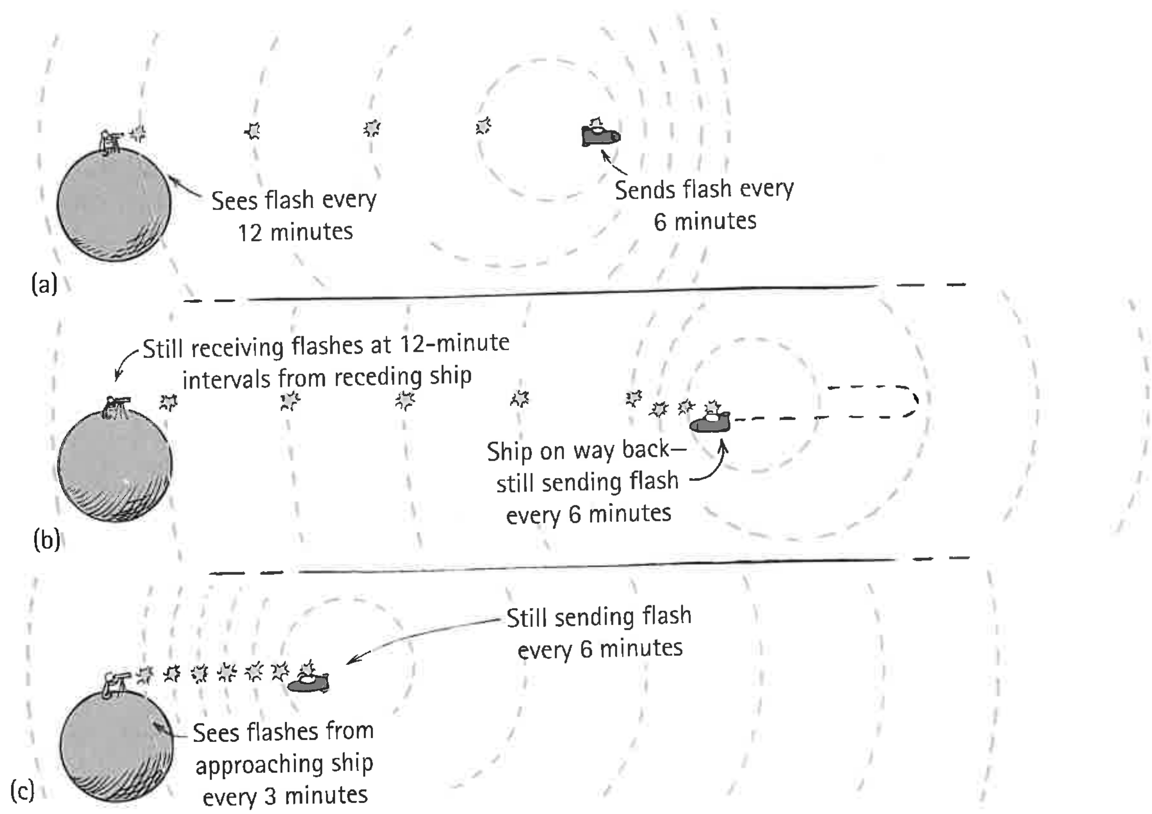


FIGURE 35.17

The spaceship emits flashes every 6 min during a 2-h trip. During the first hour, it recedes from Earth. During the second hour, it approaches Earth.

Earth, a total of 10 flashes are emitted (after the "starting gun" signal). If the ship departs from Earth at noon, clocks aboard the ship read 1 PM when the tenth flash is emitted. What time will it be on Earth when this tenth flash reaches Earth? The answer is 2 PM. Why? Because the time it takes Earth to receive 10 flashes at 12-minute intervals is 10 × (12 min), or 120 min (= 2 h).

Suppose the spaceship is somehow able to turn around "on a dime" (in a negligibly short time) and return at the same high speed. During the hour of return, it emits 10 more flashes at 6-min intervals. These flashes are received every 3 minutes on Earth, so all 10 flashes come in 30 minutes. A clock on Earth will read 2:30 PM when the spaceship completes its 2-hour trip. We see that the earthbound twin has aged half an hour more than the twin aboard the spaceship!

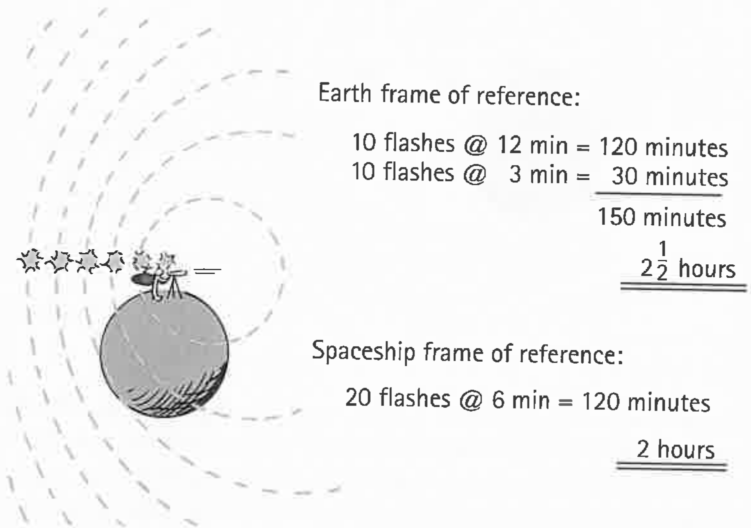


FIGURE 35.18
The trip that takes 2 h in the frame of reference of the spaceship takes 2 1/2 h in Earth's frame of reference.

The result is the same from either frame of reference. Consider the same trip again, only this time with flashes emitted from Earth at regularly spaced 6-minute intervals in Earth time. From the frame of reference of the receding spaceship, these flashes are received at 12-minute intervals (Figure 35.19a). This means that 5 flashes are seen by the spaceship during the hour of receding from Earth. During

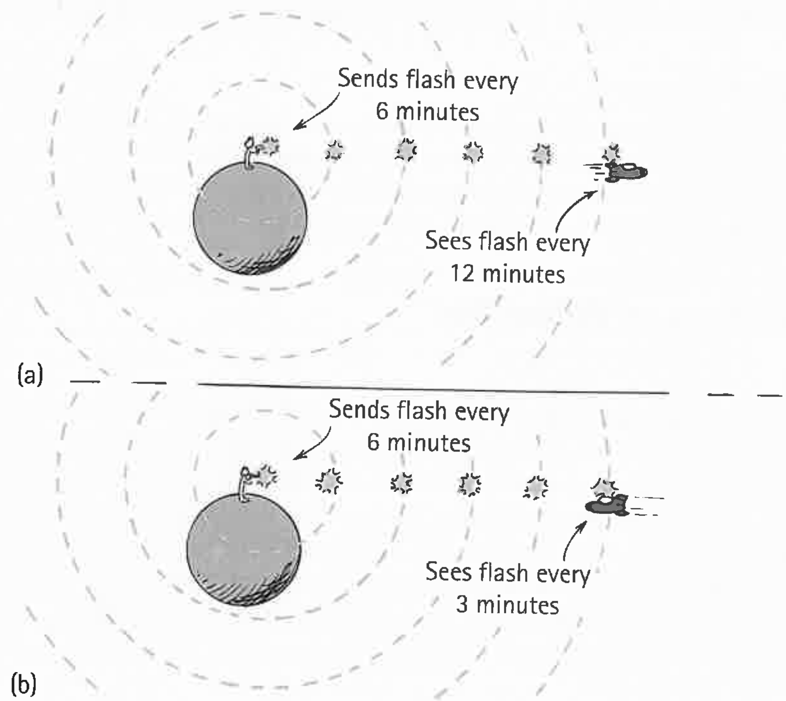


FIGURE 35.19
Flashes sent from Earth at 6-min intervals are seen at 12-min intervals by the ship when it recedes and at 3-min intervals when it approaches.

the spaceship's hour of approaching, the light flashes are seen at 3-minute intervals (Figure 35.19b), so 20 flashes will be seen.

So we see that the spaceship receives a total of 25 flashes during its 2-hour trip. According to clocks on Earth, however, the time it took to emit the 25 flashes at 6-minute intervals was 25 × (6 min), or 150 min (=2.5 h). This is shown in Figure 35.20.

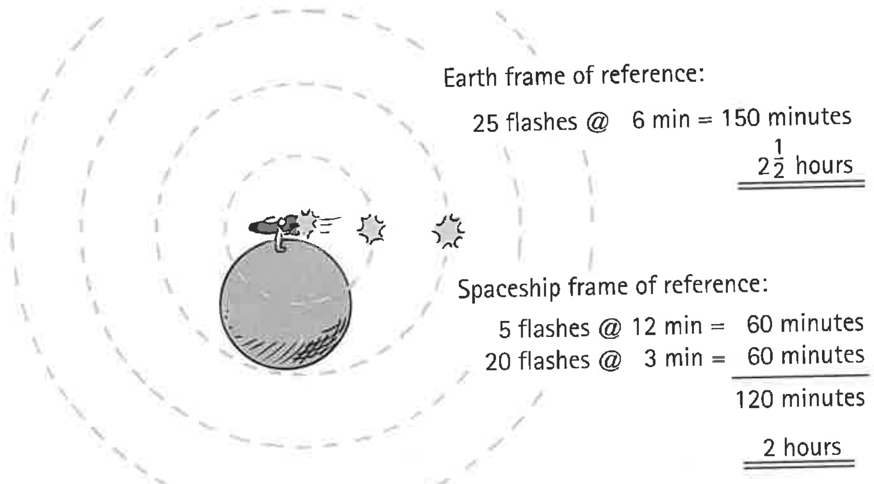


FIGURE 35.20
A time interval of 2 1/2 h on Earth is seen to take 2 h in the spaceship's frame of reference.

So both twins agree on the same results, with no dispute as to who ages more. While the stay-at-home twin remains in a single reference frame, the traveling twin has experienced two different frames of reference, separated by the acceleration of the spaceship in turning around. The spaceship has in effect experienced two different realms of time, while Earth has experienced a still different but single realm of time. The twins can meet again at the same place in space only at the expense of time.

CHECK POINT

Since motion is relative, can't we say as well that the spaceship is at rest and the Earth moves, in which case the twin on the spaceship ages more?

Check Your Answer

No, not unless Earth then undergoes the turnaround and returns, as our spaceship did in the twin-trip example. The situation is not symmetrical, for one twin remains in a single reference frame in spacetime during the trip while the other makes a distinct change of reference frame, as evidenced by the acceleration in turning around.

Addition of Velocities

Most people know that if you walk at 1 km/h along the aisle of a train that moves at 60 km/h, your speed relative to the ground is 61 km/h if you walk in the same direction as the moving train and 59 km/h if you walk in the opposite direction. What most people know is *almost* correct. Taking special relativity into account, one's speeds are *very nearly* 61 km/h and 59 km/h, respectively.

For everyday objects in uniform (nonaccelerating) motion, we ordinarily combine velocities by the simple rule

$$V = v_1 + v_2$$

But this rule does not apply to light, which always has the same velocity, c. Strictly speaking, the above rule is an approximation of the relativistic rule for adding velocities. We'll not treat the long derivation but simply state the rule:

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

The numerator of this formula makes common sense. But this simple sum of two velocities is altered by the second term in the denominator, which is significant only when both v_1 and v_2 are nearly c.

As an example, consider a spaceship moving away from you at a velocity of $0.5c$. It fires a rocket that thrusts in the same direction, also away from you, at a speed of $0.5c$ relative to itself. How fast does the rocket move relative to you? The nonrelativistic rule would say that the rocket moves at the speed of light in your reference frame. But, in fact,

$$V = \frac{0.5c + 0.5c}{1 + \frac{0.25c^2}{c^2}} = \frac{c}{1.25} = 0.8c$$

which illustrates another consequence of relativity: No material object can travel as fast as, or faster than, light.

Suppose that the spaceship instead fires a pulse of laser light in its direction of travel. How fast does the pulse move in your frame of reference?

$$V = \frac{0.5c + c}{1 + \frac{0.5c^2}{c^2}} = \frac{1.5c}{1.5} = c$$

No matter what the relative velocities between two frames, light moving at c in one frame will be seen to be moving at c in any other frame. If you try chasing light, you can never catch it.

Space Travel

One of the old arguments against the possibility of human interstellar travel was that our life span is too short. It was argued, for example, that the star nearest Earth (after the Sun), Alpha Centauri, is 4 light-years away, and a round-trip even at the speed of light would require 8 years.[3] And even a speed-of-light voyage to the center of our galaxy, 25,000 light-years distant, would require 25,000 years. But these arguments fail to take into account time dilation. Time for a person on Earth and time for a person in a high-speed spaceship are not the same.

A person's heart beats to the rhythm of the realm of spacetime in which it finds itself. And one realm of spacetime seems the same as any other to the heart, but not to an observer who stands outside the heart's frame of reference. For example, astronauts traveling at 99% of c could go to the star Procyon (10.4 light-years distant) and back in 21 Earth years. Because of time dilation, however, only 3 years would pass for the astronauts. This is what all their clocks would tell them—and, biologically, they would be only 3 years older. It would be the space officials greeting them on their return who would be 21 years older!

At higher speeds, the results are even more impressive. At a speed of 99.99% of c, travelers could travel a distance of slightly more than 70 light-years in a single year

[3] A light-year is the distance light travels in 1 year, 9.46×10^{12} km.

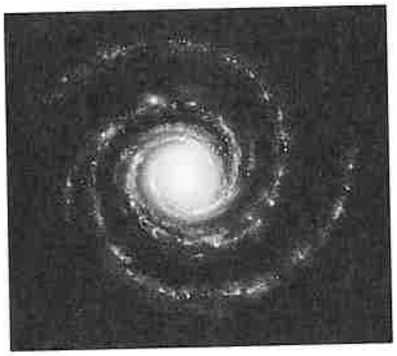


FIGURE 35.21

From the Earth frame of reference, light takes 25,000 years to travel from the center of our Milky Way galaxy to our solar system. From the frame of reference of a high-speed spaceship flying outward from the galactic center toward Earth, the trip takes less time. If a frame of reference could be attached to the light itself, the travel time could be reduced to zero.

of their own time; at 99.999% of *c*, this distance would be pushed appreciably farther than 200 light-years. A 5-year trip for them would take them farther than light travels in 1000 Earth-time years!

Present technology does not permit such journeys. Getting enough propulsive energy and shielding against radiation are both prohibitive problems. Spaceships traveling at relativistic speeds would require billions of times the energy used to put a space shuttle into orbit. Even some kind of interstellar ramjet that scooped up interstellar hydrogen gas for burning in a fusion reactor would have to overcome the enormous retarding effect of scooping up the hydrogen at high speeds. And the space travelers would encounter interstellar particles just as if they had a large particle accelerator pointed at them. No way of shielding such intense particle bombardment for prolonged periods of time is presently known. For the present, interstellar space travel must be relegated to science fiction. Not because of scientific fantasy, but simply because of the impracticality of space travel. Traveling close to the speed of light in order to take advantage of time dilation is completely consistent with the laws of physics.

We can see into the past, but we cannot go into the past. For example, we experience the past when we look at the night skies. The starlight impinging on our eyes left those stars dozens, hundreds, even millions of years ago. What we see is the stars as they were long ago. We are thus eyewitnesses to ancient history—and can only speculate about what may have happened to the stars in the interim.

If we are looking at light that left a star, say, 100 years ago, then it follows that any sighted beings in that solar system are seeing us by light that left *here* 100 years ago and that, further, if they possessed super telescopes, they might very well be able to eyewitness earthly events of a century ago—the aftermath of the American Civil War, for instance. They would see our past, but they would still see events in a forward direction; they would see our clocks running clockwise.

We can speculate about the possibility that time might just as well move counterclockwise into the past as clockwise into the future. Why is it, we might ask, that in space we can move forward or back, left or right, up or down, but we can move only in one direction through time? Quite interestingly, the mathematics of elementary-particle interactions permits "time reversal," although there are some particle interactions that slightly favor one direction in time. Hypothetical particles that can move both faster than light and backward in time are called *tachyons*. In any case, for the complex organism called a human being, time has only one direction.[4]

This conclusion is blithely ignored in a limerick that is a favorite with scientist types:

> There was a young lady named Bright
> Who traveled much faster than light.
> She departed one day
> In a relative way
> And returned on the previous night.

Even with our heads fairly well into relativity, we may still unconsciously cling to the idea that there is an absolute time and compare all these relativistic effects to it—recognizing that time changes this way and that way for this speed and that speed, yet feeling that there still is some basic or absolute time. We may tend to think that the time we experience on Earth is fundamental and that other times are not. This is understandable: We're earthlings. But the idea is confining. From the point of view of observers elsewhere in the universe, we may be moving at relativistic speeds; they see us living in slow motion. They may see us living lifetimes a hundred times as long as theirs, just as with super telescopes we would see them living lifetimes a hundredfold longer than ours. There is no universally standard time—none.

If traveling backward in time were possible, wouldn't we have tourists from the future?

[4] It has been speculated that if we moved backward through time, we wouldn't know it, for then we would remember our future and would think it was our past!

Century Hopping

Let's push our science fiction to a possible time in the future when the prohibitive problems of energy supplies and of radiation have been overcome and space travel is a routine experience. People will have the option of taking a trip and returning to any future century of their choosing. For example, one might depart from Earth in a high-speed spaceship in the year 2100, travel for 5 years or so, and return in the year 2500. One could live among the earthlings of that period for a while and depart again to try out the year 3000 for style.

People could keep jumping into the future with some expense of their own time—but they could not trip into the past. They could never return to the same era on Earth to which they had bade farewell. Time, as we know it, travels one way—forward. Here on Earth, we move constantly into the future at the steady rate of 24 hours per day. An astronaut leaving on a deep-space voyage must live with the fact that, upon return, much more time will have elapsed on Earth than the astronaut has subjectively and physically experienced during the voyage. The credo of all star travelers, whatever their physiological condition, will be permanent farewell.

We think of time and then we think of the universe. We think of the universe and we wonder about what went on before the universe began. We wonder about what will happen if the universe ceases to exist in time. But the concept of time applies to events and entities within the universe, not to the universe as a whole. Time is "in" the universe; the universe is not "in" time. Without the universe, there is no time; no before, no after. Likewise, space is "in" the universe; the universe is not "in" a region of space. There is no space "outside" the universe. Spacetime exists within the universe. Think about that!

Length Contraction

As objects move through spacetime, space as well as time changes. In a nutshell, space is contracted, making the objects look shorter when they move by us at relativistic speeds. This **length contraction** was first proposed by the physicist George F. FitzGerald and mathematically expressed by another physicist, Hendrik A. Lorentz (mentioned earlier). Whereas these physicists hypothesized that matter contracts, Einstein saw that what contracts is space itself. Nevertheless, because Einstein's formula is the same as Lorentz's, we call the effect the *Lorentz contraction:*

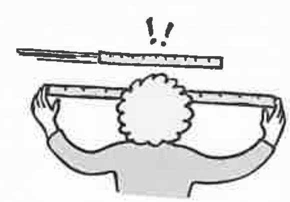


FIGURE 35.22

The Lorentz contraction. The meterstick is measured to be half as long when traveling at 87% of the speed of light relative to the observer.

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

where v is the relative velocity between the observed object and the observer, c is the speed of light, L is the measured length of the moving object, and L_0 is the measured length of the object at rest.[5]

Suppose that an object is at rest so that $v = 0$. When we substitute $v = 0$ in the Lorentz equation, we find $L = L_0$, as we would expect. When we substitute various large values of v in the Lorentz equation, we begin to see the calculated L get smaller and smaller. At 87% of c, an object would be contracted to half its original length. At 99.5% of c, it would contract to one-tenth its original length. If the object were somehow able to move at c, its length would be zero. This is one of the reasons we say that the speed of light is the upper limit for the speed of any moving object. Another limerick popular with the science heads is this one:

[5]We can express this as $L = \frac{1}{\gamma} L_0$. Where $\frac{1}{\gamma}$ is always 1 or less (because γ is always 1 or greater). Note that we do not explain how the length-contraction equation or other equations come about. We simply state equations as "guides to thinking" about the ideas of special relativity.

Time dilation: Moving clocks run slowly. Length contraction: Moving objects are shorter (in the direction of motion).

There was a young fencer named Fisk,
Whose thrust was exceedingly brisk.
So fast was his action
The Lorentz contraction
Reduced his rapier to a disk.

As Figure 35.23 indicates, contraction takes place only in the direction of motion. If an object is moving horizontally, no contraction takes place vertically.

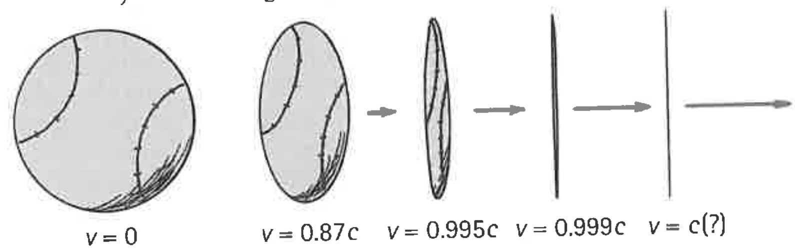


FIGURE 35.23

As speed increases, length in the direction of motion decreases. Lengths in the perpendicular direction do not change.

Length contraction should be of considerable interest to space voyagers. The center of our Milky Way galaxy is 25,000 light-years away. Does this mean that if we traveled in that direction at the speed of light, it would take 25,000 years to get there? From an Earth frame of reference, yes; but to the space voyagers, decidedly not! At the speed of light, the 25,000 light-year distance would be contracted to no distance at all. The imagined space voyagers would arrive there instantly!

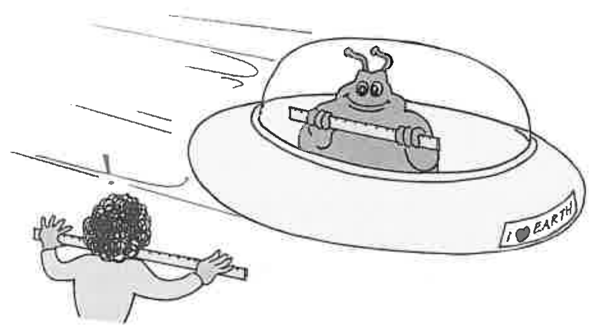


FIGURE 35.24

INTERACTIVE FIGURE

In the frame of reference of our meterstick, its length is 1 m. Observers in a moving frame see *our* metersticks contracted, while we see *their* metersticks contracted. The effects of relativity are always attributed to "the other guy."

For hypothetical travel near the speed of light, length contraction and time dilation are just two faces of the same phenomenon. If astronauts go so fast that they find the distance to the nearest star to be just 1 light-year instead of the 4 light-years measured from Earth, they make the trip in a little more than 1 year. But observers back on Earth say that the clocks aboard the spaceship have slowed so much that they tick off only 1 year in 4 years of Earth time. Both agree on what happens: The astronauts are only a little more than a year older when they reach the star. One set of observers say it's because of length contraction; the other set say it's because of time dilation. Both are right.

If space voyagers are ever able to boost themselves to relativistic speeds, they will find distant parts of the universe drawn closer by space contraction, while observers back on Earth will see the astronauts covering more distance because they age more slowly.

CHECK POINT

A rectangular billboard in space has the dimensions 10 m × 20 m. How fast, and in what direction with respect to the billboard, would a space traveler have to pass for the billboard to appear square?

Check Your Answer

The space traveler would have to travel at 0.87*c* in a direction parallel to the longer side of the board.

Relativistic Momentum

Recall our study of momentum in Chapter 6. We learned that the change of momentum mv of an object is equal to the impulse Ft applied to it: $Ft = \Delta mv$, or, $Ft = \Delta p$, where $p = mv$. If you apply more impulse to an object that is free to move, the object acquires more momentum. Double the impulse, and the momentum doubles. Apply 10 times the impulse, and the object gains 10 times as much momentum. Does this mean that momentum can increase without any limit? The answer is *yes*. Does this mean that speed can also increase without any limit? The answer is *no!* Nature's speed limit for material objects is c.

To Newton, infinite momentum would mean infinite mass or infinite speed. But not so in relativity. Einstein showed that a new definition of momentum is required. It is

$$p = \gamma mv$$

where γ is the Lorentz factor (recall that γ is always 1 or greater). This generalized definition of momentum is valid in all uniformly moving reference frames. *Relativistic momentum* is larger than mv by a factor of γ. For everyday speeds much less than c, γ is nearly equal to 1, so p is nearly equal to mv. Newton's definition of momentum is valid at low speeds. At higher speeds, γ grows dramatically, and so does relativistic momentum. As speed approaches c, γ approaches infinity! No matter how close to c an object is pushed, it would still require infinite impulse to give it the last bit of speed needed to reach c—clearly impossible. Hence we see that no body with mass can be pushed to the speed of light, much less beyond it.

FIGURE 35.25
The Stanford Linear Accelerator is 3.2 km (2 mi) long. But to electrons moving through it at $0.99999999995c$, the accelerator is only 3.2 cm long. The electrons start their journey in the foreground, and they smash into targets, or are otherwise studied, in the experimental areas beyond the freeway (near the top of the photo).

Subatomic particles are routinely pushed to nearly the speed of light. The momenta of such particles may be thousands of times more than the Newtonian expression mv predicts. Classically, the particles behave as if their masses increase with speed. Einstein initially favored this interpretation and later changed his mind to keep mass a constant, a property of matter that is the same in all frames of reference. So it is γ that changes with speed, not mass. The increased momentum of a high-speed particle is evident in the increased "stiffness" of its trajectory. The more momentum it has, the "stiffer" is its trajectory and the harder it is to deflect.

We see this when a beam of electrons is directed into a magnetic field. Charged particles moving in a magnetic field experience a force that deflects them from their normal paths. For small momentum, the path curves sharply. For large momentum, there is greater stiffness and the path curves only a little (Figure 35.26). Even though one particle may be moving only a little faster than another one—say, 99.9% of the speed of light instead of 99% of the speed of light—its momentum will be considerably greater and it will follow a straighter path in the magnetic field. This stiffness must be compensated for in circular accelerators like cyclotrons and synchrotrons, where momentum dictates the radius of curvature. In the linear accelerator shown in Figure 35.25, the particle beam travels in a straight-line path and momentum changes don't produce deviations from a straight-line path. Deviations occur when the beam of electrons is bent at the exit port by magnets, as indicated in Figure 35.26. Whatever the type of particle accelerator, physicists working with subatomic particles every day verify the correctness of the relativistic definition of momentum and the speed limit imposed by nature.

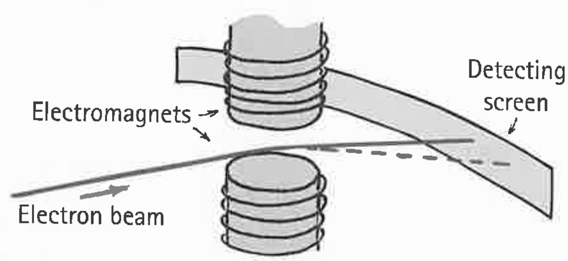


FIGURE 35.26
If the momentum of the electrons were equal to the Newtonian value mv, the beam would follow the dashed line. But because the relativistic momentum γmv is greater, the beam follows the "stiffer" trajectory shown by the solid line.

To summarize, we see that, as the speed of an object approaches the speed of light, its momentum approaches infinity—which means there is no way that the speed of light can be reached. There is, however, at least one thing that reaches the speed of light—light itself! But the photons of light are massless, and the equations that apply to them are different. Light travels always at the same speed. So, interestingly, a material particle can never be brought to the speed of light, and light can never be brought to rest.

Mass, Energy, and $E = mc^2$

Einstein linked not only space and time but also mass and energy. A piece of matter, even at rest and not interacting with anything else, has an "energy of being." This is called its *rest energy*. Einstein concluded that it takes energy to make mass and that energy is released if mass disappears. The amount of energy E is related to the amount of mass m by the most celebrated equation of the 20th century:

$$E = mc^2$$

The c^2 is the conversion factor between energy units and mass units. Because of the large magnitude of c, a small mass corresponds to an enormous quantity of energy.[6]

FIGURE 35.27
Saying that a power plant delivers 90 million megajoules of energy to its consumers is equivalent to saying that it delivers 1 gram of energy to its consumers, because mass and energy are equivalent.

Recall from the previous chapter that tiny decreases of nuclear mass in both nuclear fission and nuclear fusion produce enormous releases of energy, all in accord with $E = mc^2$. To the general public, $E = mc^2$ is synonymous with nuclear energy. If we were to weigh a fully fueled nuclear power plant, then weigh it again a week later, we'd find it weighs slightly less—about 1 gram less for every kilogram of fuel that underwent fission in that week. Part of the fuel's mass has been converted to energy. Now, interestingly enough, if we were to weigh a coal-burning power plant and all the coal and oxygen it consumes in a week, and then weigh it again with all the carbon dioxide and other combustion products that came out during the week, we'd also find it all weighs slightly less. Again, mass has been converted to energy. About 1 part in a billion has been converted. Get this: If both plants produce the same amount of energy, the mass change will be the same for both—whether energy is released by nuclear or chemical mass conversion makes no difference. The chief difference lies in the amount of energy released in each individual reaction and the amount of mass involved. Fissioning of a single uranium nucleus releases 10 million times as much energy as the combustion of carbon to produce a single carbon dioxide molecule. Hence, a few truckloads of uranium fuel will power a fission plant while a coal-burning plant consumes many hundred car trainloads of coal.

When we strike a match, phosphorus atoms in the match head rearrange themselves and combine with oxygen in the air to form new molecules. The resulting molecules have very slightly less mass than the separate phosphorus and oxygen molecules. From a mass standpoint, the whole is slightly less than the sum of its parts, by amounts that escape our notice. For all chemical reactions that give off energy, there is a corresponding decrease in mass of about 1 part in a billion.

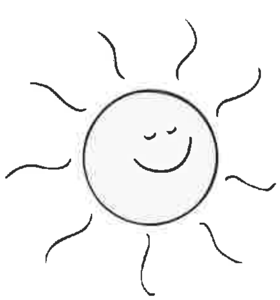


FIGURE 35.28
In 1 second, 4.5 million tons of mass are converted to radiant energy in the Sun. The Sun is so massive, however, that in 1 million years only 1 ten-millionth of the Sun's mass will have been converted to radiant energy.

For nuclear reactions, a decrease in mass by 1 part in a thousand can be directly measured by a variety of devices. This decrease of mass in the Sun by the process of thermonuclear fusion bathes the solar system with radiant energy and nourishes life. The present stage of thermonuclear fusion in the Sun has been going on for the past 5 billion years, and there is sufficient hydrogen fuel for fusion to last another 5 billion years. It is nice to have such a big Sun!

The equation $E = mc^2$ is not restricted to chemical and nuclear reactions. A change in energy of any object at rest is accompanied by a change in its mass. The filament of a lightbulb energized with electricity has more mass than when it is turned off. A hot cup of tea has more mass than the same cup of tea when cold. A wound-up spring clock has more mass than the same clock when unwound. But these examples involve incredibly small changes in mass—far too small to be measured. Even the much larger changes of mass in radioactive change were not

[6]When c is in meters per second and m is in kilograms, then E will be in joules. If the equivalence of mass and energy had been understood long ago when physics concepts were first being formulated, there would probably be no separate units for mass and energy. Furthermore, with a redefinition of space and time units, c could equal 1, and $E = mc^2$ would simply be $E = m$.

measured until after Einstein predicted the mass–energy equivalence. Now, however, mass-to-energy and energy-to-mass conversions are measured routinely.

Consider a coin with a mass of 1 g. You'd expect 2 of the same coins to have a mass of 2 g, 10 coins to have a mass of 10 g, and 1,000 coins piled in a box to have a mass of 1 kg. Not so if the coins attract or repel each other. Suppose, for example, that each coin carries a negative electric charge so that each coin repels all other coins. Then forcing them together in the box takes work. This work adds to the mass of the collection. So a box containing 1,000 negatively charged coins has more than 1 kg of mass. If, on the other hand, the coins all attracted one another (as nucleons in the nucleus attract one another), it takes work to separate them; then a box of 1,000 coins would have a mass less than 1 kg. So the mass of an object is not necessarily equal to the sum of the masses of its parts, as we know from measuring the masses of nuclei. The effect would be dramatically enormous if we could deal with bare charged particles. If we could force together a number of electrons whose masses add separately to 1 g into a 10-cm-diameter sphere, the collection would have a mass of 40 billion kg! The equivalence of mass and energy is indeed profound.

Before physicists came to understand that the electron is a fundamental particle with no measurable radius, some speculated that it had a certain size and that its mass is merely a measure of how much work was required to compress its charge to that size.[7]

In ordinary units of measurement, the speed of light c is a large quantity and its square is even larger—hence a small amount of mass stores a large amount of energy. The quantity c^2 is a "conversion factor." It converts the measurement of mass to the measurement of equivalent energy. Or it is the ratio of rest energy to mass; $E/m = c^2$. Its appearance in either form of this equation has nothing to do with light and nothing to do with motion. The magnitude of c^2 is 90 quadrillion (9×10^{16}) J/kg. One kilogram of matter has an "energy of being" equal to 90 quadrillion J. Even a speck of matter with a mass of only 1 mg has a rest energy of 90 billion J.

The equation $E = mc^2$ is more than a formula for the conversion of mass into other kinds of energy, or vice versa. It states even more: that energy and mass are the *same thing*. Mass is congealed energy. If you want to know how much energy is in a system, measure its mass. For an object at rest, its energy *is* its mass. Energy, like mass, exhibits inertia. Shake a massive object back and forth; it is energy itself that is hard to shake.

$E = mc^2$ says that energy and mass are related. Mass is congealed energy.

The first evidence for the conversion of radiant energy to mass was provided in 1932 by the American physicist Carl Anderson. He discovered the *positron* by the track it left in a cloud chamber. The positron is the *antiparticle* of the electron, equal in mass and spin to the electron but opposite in charge. When a high-frequency photon comes close to an atomic nucleus, it can create an electron and a positron together as a pair, thus creating mass. The created particles fly apart. The positron is not part of normal matter because it lives such a short time. As soon as it encounters an electron, the pair is annihilated, sending out two gamma rays in the process. Then mass is converted back to radiant energy.[8]

[7]San Francisco Sidewalk Astronomer John Dobson speculates that, just as a clock becomes more massive when we do work on it by winding it against the resistance of its spring, the mass of the entire universe is nothing more than the energy that has gone into winding it up against mutual gravitation. In this view, the mass of the universe is equivalent to the work done in spreading it out.

[8]Recall that the energy of a photon is $E = hf$ and that the mass energy of a particle is $E = mc^2$. High-frequency photons routinely convert their energy to mass when they produce pairs of particles in nature—and in accelerators, where the processes can be observed. Why pairs? Mainly because that's the only way the conservation of charge is not violated. So, when an electron is created, an antiparticle positron is created also. Equating the two equations, $hf = 2mc^2$, where m is the mass of a particle (or antiparticle), we see the minimum frequency of a gamma ray for the production of a particle pair is $f = 2mc^2/h$.

CHECK POINT

Can we look at the equation $E = mc^2$ in another way and say that matter transforms into pure energy when it is traveling at the speed of light squared?

Check Your Answer

No, no, no! Matter cannot be made to move at the speed of light, let alone the speed of light squared (which is not a speed!). The equation $E = mc^2$ simply means that energy and mass are "two sides of the same coin."

The Correspondence Principle

We introduced the correspondence principle in Chapter 32. Recall that it states that any new theory or any new description of nature must agree with the old where the old gives correct results. If the equations of special relativity are valid, they must correspond to those of classical mechanics when speeds much less than the speed of light are considered.

How nice that Einstein's equations for time, length, and momentum correspond to their classical expressions for everyday speeds.

The relativity equations for time, length, and momentum are

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma t_0$$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = L_0/\gamma$$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma mv$$

Note that these equations each reduce to Newtonian values for speeds that are very small compared with c. Then the ratio v^2/c^2 is very small and, for everyday speeds, may be taken to be zero. The relativity equations become

$$t = \frac{t_0}{\sqrt{1 - 0}} = t_0$$

$$L = L_0\sqrt{1 - 0} = L_0$$

$$p = \frac{mv}{\sqrt{1 - 0}} = mv$$

So, for everyday speeds, the momentum, length, and time of moving objects are essentially unchanged. The equations of special relativity hold for all speeds, although they differ appreciably from classical equations only for speeds near the speed of light.

Einstein's theory of relativity has raised many philosophical questions. What, exactly, is time? Can we say that it is nature's way of seeing to it that everything does not all happen at once? And why does time seem to move in one direction? Has it always moved *forward?* Are there other parts of the universe in which time moves *backward?* Is it likely that our three-dimensional perception of a four-dimensional world is only a beginning? Could there be a fifth dimension? A sixth dimension? A seventh dimension? And, if so, what would the nature of these dimensions be? Perhaps these unanswered questions will be answered by the physicists of tomorrow. How exciting!

SUMMARY OF TERMS

Frame of reference A vantage point (usually a set of coordinate axes) with respect to which position and motion may be described.

Postulates of the special theory of relativity (1) All laws of nature are the same in all uniformly moving frames of reference. (2) The speed of light in free space has the same measured value regardless of the motion of the source or the motion of the observer; that is, the speed of light is a constant.

Simultaneity Occurring at the same time. Two events that are simultaneous in one frame of reference need not be simultaneous in a frame moving relative to the first frame.

Spacetime The four-dimensional continuum in which all events take place and all things exist: Three dimensions are the coordinates of space, and the fourth is time.

Time dilation The slowing of time as a result of speed.

Length contraction The contraction of space in an observer's direction of motion as a result of speed.

REVIEW QUESTIONS

Motion Is Relative

1. If you walk at 1 km/h down the aisle of a train that moves at 60 km/h, what is your speed relative to the ground?

2. In the previous question, is your approximate speed relative to the Sun as you walk down the aisle of the train slightly more or very much more?

Michelson–Morley Experiment

3. What hypothesis did G. F. FitzGerald make to explain the findings of Michelson and Morley?

4. What classical idea about space and time was rejected by Einstein?

Postulates of the Special Theory of Relativity

5. Cite two examples of Einstein's first postulate.

6. Cite one example of Einstein's second postulate.

Simultaneity

7. Inside the moving compartment of Figure 35.4, light travels a certain distance to the front end and a certain distance to the back end of the compartment. How do these distances compare as seen in the frame of reference of the moving rocket?

8. How do the distances in Question 7 compare as seen in the frame of reference of an observer on a stationary planet?

Spacetime

9. How many coordinate axes are usually used to describe three-dimensional space? What does the fourth dimension measure?

10. Under what condition will you and a friend share the same realm of spacetime? When will you not share the same realm?

11. What is special about the ratio of the distance traveled by a flash of light and the time the light takes to travel this distance?

Time Dilation

12. Time is required for light to travel along a path from one point to another. If this path is seen to be longer because of motion, what happens to the time it takes for light to travel this longer path?

13. What do we call the "stretching out" of time?

14. What is an algebraic expression for the Lorentz factor γ (gamma)? Why is γ never less than 1?

15. How do measurements of time differ for events in a frame of reference that moves at 50% the speed of light relative to us? At 99.5% the speed of light relative to us?

16. What is the evidence for time dilation?

The Twin Trip

17. When a flashing light approaches you, each flash that reaches you has a shorter distance to travel. What effect does this have on how frequently you receive the flashes?

18. When a flashing light source approaches you, does the speed of light or the frequency of light—or both—increase?

19. If a flashing light source moves toward you fast enough so that the duration between flashes is half as long, how long will be the duration between flashes if the source is moving away from you at the same speed?

20. How many frames of reference does the stay-at-home twin experience in the twin trip? How many frames of reference does the traveling twin experience?

Addition of Velocities

21. What is the maximum value of $v_1 v_2/c^2$ in an extreme situation? What is the smallest value?

22. Is the relativistic rule

$$V = \frac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{c^2}}$$

consistent with the fact that light can have only one speed in all uniformly moving reference frames?

Space Travel

23. What two main obstacles prevent us from traveling today throughout the galaxy at relativistic speeds?

24. What is the universal standard of time?

Length Contraction

25. How long would a meterstick appear to be if it were traveling like a properly thrown spear at 99.5% the speed of light?

26. How long would the meterstick in the previous question appear to be if it were traveling with its length perpendicular to its direction of motion? (Why is your answer different from your answer to the previous question?)

27. If you were traveling in a high-speed rocket ship, would metersticks on board appear to you to be contracted? Defend your answer.

Relativistic Momentum

28. What would be the momentum of an object pushed to the speed of light?

29. When a beam of charged particles moves through a magnetic field, what is the evidence that particles in the beam have momenta greater than the value *mv*?

Mass, Energy, and $E = mc^2$

30. Compare the amount of mass converted to energy in nuclear reactions and in chemical reactions.

31. How does the energy from the fissioning of a single uranium nucleus compare with the energy from the combustion of a single carbon atom?

32. Does the equation $E = mc^2$ apply only to nuclear and chemical reactions?

33. What is the evidence for $E = mc^2$ in cosmic-ray investigations?

The Correspondence Principle

34. How does the correspondence principle relate to special relativity?

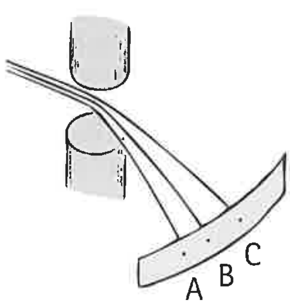
35. Do the relativity equations for time, length, and momentum hold true for everyday speeds? Explain.

PROJECT

Text Grandma and explain how Einstein's theories of relativity concern the fast and the big—that relativity is not only "out there" but that it affects this world. Tell her how these ideas stimulate your quest for more knowledge about the universe. Impress Grandma by proper use of the words *there, they're,* and *their* in your text.

RANKING

1. Electrons are fired at different speeds through a magnetic field and are bent from their straight-line paths to hit the detector at the points shown. Rank the speeds of the electrons from highest to lowest.



2. To an Earth observer, metersticks on three spaceships are seen to have these lengths. Rank the speeds of the spaceships relative to Earth from highest to lowest.

EXERCISES

1. The idea that force causes acceleration doesn't seem strange. This and other ideas of Newtonian mechanics are consistent with our everyday experience. But the ideas of relativity do seem odd and more difficult to grasp. Why is this?

2. If you were in a smooth-riding train with no windows, could you sense the difference between uniform motion and rest? Between accelerated motion and rest? Explain how you could make such a distinction with a bowl filled with water.

3. A person riding on the roof of a freight train throws a ball forward. (a) Neglecting air drag and relative to the ground, is the ball moving faster or slower when the train is moving than when it is standing still? (b) Relative to the freight car, is the ball moving faster or slower when the train is moving than when the train is standing still?

4. Suppose instead that the person riding on top of the freight car shines a searchlight beam in the direction in which the train is traveling. Compare the speed of the light beam relative to the ground when the train is at rest and when it is in motion. How does the behavior of the light beam differ from the behavior of the ball in Exercise 3?
5. Why did Michelson and Morley at first consider their experiment a failure? (Have you ever encountered other examples where failure has to do not with the lack of ability, but with the impossibility of the task?)
6. When you drive down the highway, you are moving through space. What else are you moving through?
7. In Chapter 26, we learned that light travels more slowly in glass than in air. Does this contradict Einstein's second postulate?
8. Astronomers view light coming from distant galaxies moving away from Earth at speeds greater than 10% the speed of light. How fast does this light meet the telescopes of the astronomers?
9. Does special relativity allow *anything* to travel faster than light? Explain.
10. When a light beam approaches you, its frequency is greater and its wavelength less. Does this contradict the postulate that the speed of light cannot change? Defend your answer.
11. The beam of light from a laser on a rotating turntable casts into space. At some distance, the beam moves across space faster than c. Why does this not contradict relativity?
12. Can an electron beam sweep across the face of a cathode-ray tube at a speed greater than the speed of light? Explain.
13. Consider the speed of the point where scissors blades meet when the scissors are closed. The closer the blades are to being closed, the faster the point moves. The point could, in principle, move faster than light. Likewise for the speed of the point where an ax meets wood when the ax blade meets the wood not quite horizontally; the contact point travels faster than the ax. Similarly, a pair of laser beams that are crossed and moved toward being parallel produce a point of intersection that can move faster than light. Why do these examples not contradict special relativity?
14. If two lightning bolts hit exactly the same place at exactly the same time in one frame of reference, is it possible that observers in other frames will see the bolts hitting at different times or at different places?
15. Event A occurs before event B in a certain frame of reference. How could event B occur before event A in some other frame of reference?
16. Suppose that the lightbulb in the rocket ship in Figures 35.4 and 35.5 is closer to the front than to the rear of the compartment so that the observer in the ship sees the light reaching the front before it reaches the back. Is it still possible that the outside observer will see the light reaching the back first?
17. The speed of light is a speed limit in the universe—at least for the four-dimensional universe we comprehend. No material particle can attain or surpass this limit even when a continuous, unremitting force is exerted on it. What evidence supports this?
18. Since there is an upper limit on the speed of a particle, does it follow that there is also an upper limit on its momentum, and, therefore, on its kinetic energy? Explain.
19. Light travels a certain distance in, say, 20,000 years. How is it possible that an astronaut, traveling slower than light, could go as far in 20 years of her life as light travels in 20,000 years?
20. Is it possible in principle for a human being who has a life expectancy of 70 years to make a round-trip journey to a part of the universe thousands of light-years distant? Explain.
21. A twin who makes a long trip at relativistic speeds returns younger than her stay-at-home twin sister. Could she return before her twin sister was born? Defend your answer.
22. Is it possible for a son or daughter to be biologically older than his or her parents? Explain.
23. If you were in a rocket ship traveling away from Earth at a speed close to the speed of light, what changes would you note in your pulse? In your volume? Explain.
24. If you were on Earth monitoring a person in a rocket ship traveling away from Earth at a speed close to the speed of light, what changes would you note in his pulse? In his volume? Explain.
25. Due to length contraction, you see people in a spaceship passing by you as being slightly narrower than they normally appear. How do these people view you?
26. Because of time dilation, you observe the hands of your friend's watch to be moving slowly. How does your friend view your watch—as running slowly, running rapidly, or neither?
27. Does the equation for time dilation show dilation occurring for all speeds, whether slow or fast? Explain.
28. If you lived in a world where people regularly traveled at speeds near the speed of light, why would it be risky to make a dental appointment for 10:00 AM next Thursday?
29. How do the measured densities of a body compare at rest and in motion?
30. If stationary observers measure the shape of a passing object to be exactly circular, what is the shape of the object according to observers on board the object, traveling with it?
31. The formula relating speed, frequency, and wavelength of electromagnetic waves, $v = f\lambda$, was known before relativity was developed. Relativity has not changed this equation, but it has added a new feature to it. What is that feature?
32. Light is reflected from a moving mirror. How is the reflected light different from the incident light, and how is it the same?
33. As a meterstick moves past you, your measurements show its momentum to be twice its classical momentum and its length to be 1 m. In what direction is the stick pointing?
34. In the preceding exercise, if the stick is moving in a direction along its length (like a properly thrown spear), how long will you measure its length to be?
35. If a high-speed spaceship appears shrunken to half its normal length, how does its momentum compare with the classical formula $p = mv$?
36. How can the momentum of a particle increase by 5% with only a 1% increase in speed?

37. The 2-mile linear accelerator at Stanford University in California "appears" to be less than a meter long to the electrons that travel in it. Explain.
38. Electrons end their trip in the Stanford accelerator with an energy thousands of times greater than their initial rest energy. In theory, if you could travel with them, would you notice an increase in their energy? In their momentum? In your moving frame of reference, what would be the approximate speed of the target they are about to hit?
39. Two safety pins, identical except that one is latched and one is unlatched, are placed in identical acid baths. After the pins are dissolved, what, if anything, is different about the two acid baths?
40. A chunk of radioactive material encased in an idealized, perfectly insulating blanket gets warmer as its nuclei decay and release energy. Does the mass of the radioactive material and the blanket change? If so, does it increase or decrease?
41. The electrons that illuminate the screen in the picture tube of yesterday's TV sets travel at nearly one-fourth the speed of light and possess nearly 3% more energy than hypothetical nonrelativistic electrons traveling at the same speed. Does this relativistic effect tend to increase or decrease the electric bill?
42. Muons are elementary particles that are formed high in the atmosphere by the interactions of cosmic rays with atomic nuclei up there. Muons are radioactive and have average lifetimes of about two-millionths of a second. Even though they travel at almost the speed of light, very few should be detected at sea level after traveling through the atmosphere—at least according to classical physics. Laboratory measurements, however, show that muons in great number do reach Earth's surface. What is the explanation?
43. How might the idea of the correspondence principle be applied outside the field of physics?
44. What does the equation $E = mc^2$ mean?
45. According to $E = mc^2$, how does the amount of energy in a kilogram of feathers compare with the amount of energy in a kilogram of iron?
46. Does a fully charged flashlight battery weigh more than the same battery when dead? Defend your answer.
47. When we look out into the universe, we see into the past. John Dobson, founder of the San Francisco Sidewalk Astronomers, says that we cannot even see the backs of our own hands *now*—in fact, we can't see anything *now*. Do you agree? Explain.
48. One of the fads of the future might be "century hopping," where occupants of high-speed spaceships would depart from Earth for several years and return centuries later. What are the present-day obstacles to such a practice?
49. Is the statement by the philosopher Kierkegaard that "Life can only be understood backwards; but it must be lived forwards" consistent with the theory of special relativity?
50. Make up four multiple-choice questions, one each that would check a classmate's understanding of (a) time dilation, (b) length contraction, (c) relativistic momentum, and (d) $E = mc^2$.

PROBLEMS

Recall, from this chapter, that the factor gamma (γ) governs both time dilation and length contraction, where

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

When you multiply the time in a moving frame by γ, you get the longer (dilated) time in your fixed frame. When you divide the length in a moving frame by γ, you get the shorter (contracted) length in your fixed frame.

1. Consider a high-speed rocket ship equipped with a flashing light source. If the frequency of flashes seen on an approaching ship is twice what it was when the ship was a fixed distance away, by how much is the period (time interval between flashes) changed? Is this period constant for a constant relative speed? For accelerated motion? Defend your answer.
2. A starship passes Earth at 80% of the speed of light and sends a drone ship forward at half the speed of light relative to itself. Show that the drone travels at 93% the speed of light relative to Earth.
3. Pretend that the starship in the previous problem is somehow traveling at c with respect to Earth and it fires a drone forward at speed c with respect to itself. Use the equation for the relativistic addition of velocities to show that the speed of the drone with respect to Earth is still c.
4. A passenger on an interplanetary express bus traveling at $v = 0.99c$ takes a 5-minute catnap, according to her watch. Show that her catnap from the vantage point of a fixed planet lasts 35 minutes.
5. According to Newtonian mechanics, the momentum of the bus in the preceding problem is $p = mv$. According to relativity, it is $p = \gamma mv$. How does the actual momentum of the bus moving at $0.99c$ compare with the momentum it would have if classical mechanics were valid? How does the momentum of an electron traveling at $0.99c$ compare with its classical momentum?
6. The bus in the previous problems is 70 feet long, according to its passengers and driver. Show that its length is seen as slightly less than 10 feet from a vantage point on a fixed planet.
7. If the bus in Problem 4 were to slow to a "mere" 10% of the speed of light, show that you would measure the passenger's catnap to last slightly more than 5 minutes.
8. If the bus driver in Problem 4 decided to drive at 99.99% of the speed of light in order to gain some time, show that you'd measure the length of the bus to be a little less than 1 foot.

9. Assume that rocket taxis of the future move about the solar system at half the speed of light. For a 1-hour trip as measured by a clock in the taxi, a driver is paid 10 stellars. The taxi-driver's union demands that pay be based on Earth time instead of taxi time. If their demand is met, show that the new payment for the same trip would be 11.5 stellars.

10. The fractional change of reacting mass to energy in a fission reactor is about 0.1%, or 1 part in a thousand. For each kilogram of uranium that undergoes fission, how much energy is released? If energy costs 3 cents per megajoule, how much is this energy worth in dollars?

CHAPTER 35 ONLINE RESOURCES

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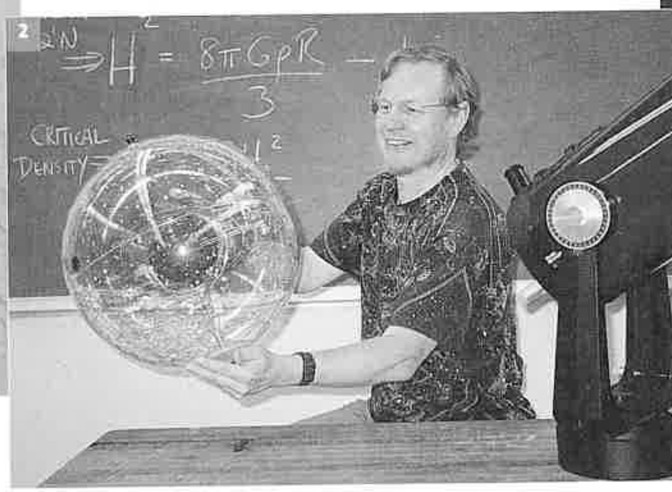
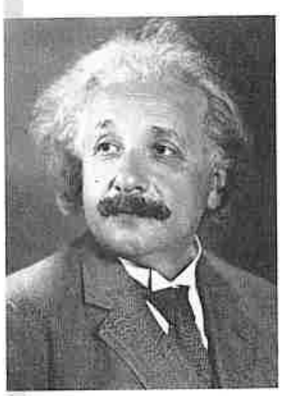
36 General Theory of Relativity



1 When your Global Positioning System (GPS) unit tells you where you are, thank Einstein. 2 Richard Crowe kicks off a lecture on general relativity with a celestial sphere. 3 When he and other astrophysicists make measurements of far-off galactic events, they also thank Einstein.

When Einstein was a young student he cut many lectures, preferring to study on his own, and in 1900 he succeeded in passing his examinations by cramming with the help of a friend's meticulous notes. He said later of this, ". . . after I had passed the final examination, I found the consideration of any scientific problem distasteful to me for an entire year." During this year he became a citizen of Switzerland; he accepted a temporary summer teaching position and tutored two young high school students. He advised their father, a high school teacher himself, to remove the boys from school, where, he maintained, their natural curiosity was being destroyed. Einstein's job as a tutor was short-lived.



It was not until two years after graduation that he got a steady job, as a patent examiner at the Swiss Patent Office in Berne. Einstein held this position for over seven years. He found the work rather interesting, sometimes stimulating his scientific imagination, but mainly freeing him of financial worries while providing time to ponder the problems in physics that puzzled him.

With no academic connections whatsoever, and with essentially no contact with other physicists, he laid out the main lines along which 20th-century theoretical physics has developed. In 1905, at the age of 26, he earned his Ph.D. in physics and published four major papers. The first was on the quantum theory of light, including an explanation of the photoelectric effect, for which he won the 1921 Nobel Prize in Physics. The second paper was on the statistical aspects of molecular theory and Brownian motion, a proof for the existence of atoms. His third and most famous paper was on special relativity. In a follow-up fourth paper he presented the famous $E = mc^2$.

Then came ten years of intense work, leading in 1915 to the **general theory of relativity** in which Einstein presented a new theory of gravitation that included Newton's theory as a special case. These trailblazing papers have greatly affected the course of modern physics.

Einstein's concerns were not limited to physics. He lived in Berlin during World War I and denounced the German militarism of his time. He publicly expressed his deeply felt conviction that warfare should be abolished and an international organization founded to govern disputes between nations. In 1933, while Einstein was visiting the United States, Hitler came to power. Einstein spoke out against Hitler's racial and political policies and resigned his position at the University of Berlin. No longer safe in Germany, Einstein came to the United States and accepted a research position at the Institute for Advanced Study in Princeton, New Jersey.

In 1939, one year before Einstein became an American citizen, and after German scientists fissioned the uranium atom, he was urged by several prominent Hungarian-American scientists to write the famous letter to President Roosevelt pointing out the scientific possibilities of a nuclear bomb. Einstein was a pacifist, but the thought of Hitler developing such a bomb prompted his action. The outcome was the development of the first nuclear bomb, which, ironically, was detonated on Japan after the fall of Germany.

Einstein believed that the universe is indifferent to the human condition and stated that if humanity were to continue, it must create a moral order. He intensely advocated world peace through nuclear disarmament. Nuclear bombs, Einstein remarked, had changed everything but our way of thinking.

Science philosopher C. P. Snow, who was acquainted with Einstein, in a review of *The Born–Einstein Letters, 1916–1955,* says this of him: "Einstein was the most powerful mind of the twentieth century, and one of the most powerful that ever lived. He was more than that. He was a man of enormous weight of personality, and perhaps most of all, of normal stature . . . I have met a number of people whom the world calls great; of these, he was by far, by an order of magnitude, the most impressive. He was—despite the warmth, the humanity, the touch of the comedian—the most different from other men."

Einstein was more than a great scientist; he was a man of unpretentious disposition with a deep concern for the welfare of his fellow beings. The choice of Einstein as the person of the century by *Time* magazine at the end of the 1900s was most appropriate—and noncontroversial.

Reference Frames—Nonaccelerated and Accelerated

Recall that Einstein postulated, in 1905, that no observation made inside an enclosed chamber could determine whether the chamber is at rest or moving with constant velocity; that is, no mechanical, electrical, optical, or any other physical measurement that one could perform inside a closed compartment in a smoothly riding train traveling along a straight track (or in an airplane flying through still air with the window curtains drawn) could possibly give any information as to whether the train was moving or at rest (or whether the plane was airborne or at rest on the runway). But if the track were not smooth and straight (or if the air were turbulent), the situation would be entirely different: Uniform motion would give way to accelerated motion, which would be easily noticed. Einstein's conviction that the laws of nature should be expressed in the same form in every frame of reference, accelerated as well as nonaccelerated, was the primary motivation that led him to the general theory of relativity.

fyi

- Special relativity is "special" in the sense that it deals with uniformly moving reference frames—ones that aren't accelerated. General relativity is "general" and deals also with accelerating reference frames. The general theory of relativity presents a new theory of gravity.

Principle of Equivalence

Long before there were real spaceships, Einstein could imagine himself in a vehicle far away from gravitational influences. In such a spaceship at rest or in uniform motion relative to the distant stars, he and everything within the ship would float freely; there would be no "up" and no "down." But, when the rocket motors were turned on and the ship accelerated, things would be different; phenomena similar to gravity would be observed. The wall adjacent to the rocket motors would push up against any occupants and become the floor, while the opposite wall would become the

FIGURE 36.1

Everything is weightless on the inside of a nonaccelerating spaceship far away from gravitational influences.

FIGURE 36.2
When the spaceship accelerates, an occupant inside feels "gravity."

ceiling. Occupants in the ship would be able to stand on the floor and even jump up and down. If the acceleration of the spaceship were equal to g, the occupants could well be convinced the ship was not accelerating but was at rest on the surface of Earth.

To examine this new "gravity" in an accelerating spaceship, let's consider the consequence of dropping two balls inside the spaceship, one ball of wood and the other of lead. When the balls are released, they continue to move upward side by side with the velocity the ship had at the moment of release. If the ship were moving at *constant velocity* (zero acceleration), the balls would remain suspended in the same place because they and the ship move the same distance in any given time interval. But because the ship is accelerating, the floor moves upward faster than the balls, with the result that the floor soon catches up with the balls (Figure 36.3). Both balls, regardless of their mass, meet the floor at the same time. Remembering Galileo's demonstration at the Leaning Tower of Pisa, occupants of the ship might be prone to attribute their observations to the force of gravity.

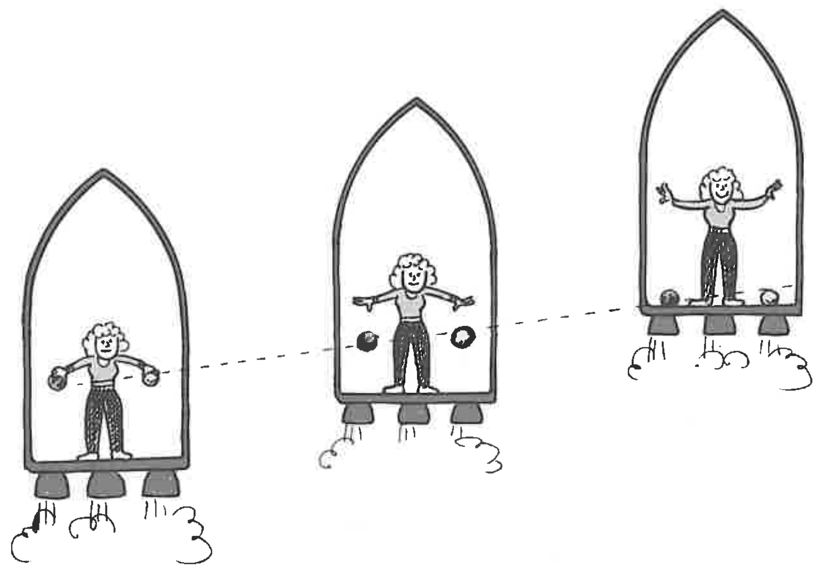


FIGURE 36.3
To an observer inside the accelerating ship, a lead ball and a wood ball appear to fall together when released.

The two interpretations of the falling balls are equally valid, and Einstein incorporated this equivalence, or impossibility of distinguishing between gravitation and acceleration, in the foundation of his general theory of relativity. The **principle of equivalence** states that observations made in an accelerated reference frame are indistinguishable from observations made in a Newtonian gravitational field. This equivalence would be interesting but not revolutionary if it could be applied only to mechanical phenomena, but Einstein went further and stated that the principle holds for all natural phenomena; it holds for optical and all electromagnetic phenomena as well.

An incorrect hypothesis, rightly treated, can sometimes produce more new useful information than unguided observation.

CHECK POINT

If you drop a ball inside a spaceship at rest on a launching pad, you'll see it accelerate to the floor. Far away from Earth, how else could you see the ball do the same?

Check Your Answer

You could also see the ball accelerate to the floor if your spaceship accelerated at g.

Bending of Light by Gravity

A ball thrown sideways in a stationary spaceship in a gravity-free region will follow a straight-line path relative both to an observer inside the ship and to a stationary observer outside the spaceship. But if the ship is accelerating, the floor overtakes the ball just as in our previous example. An observer outside the ship still sees a straight-line path, but to an observer in the accelerating ship, the path is curved; it is a parabola (Figure 36.4). The same holds true for a beam of light.

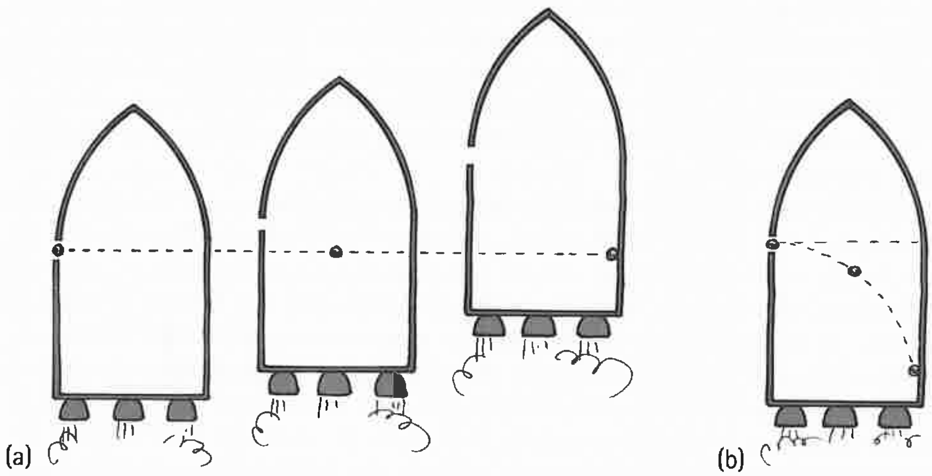


FIGURE 36.4

(a) An outside observer sees a horizontally thrown ball travel in a straight line. Because the ship is moving upward while the ball travels horizontally, the ball strikes the wall below a point opposite the window. (b) To an inside observer, the ball bends as if in a gravitational field.

Imagine that a light ray enters the spaceship horizontally through a side window, passes through a sheet of glass in the middle of the cabin, leaving a visible trace, and then reaches the opposite wall, all in a very short time. The outside observer sees that the light ray enters the window and moves horizontally along a straight line with constant velocity toward the opposite wall. But the spaceship is accelerating upward. During the time it takes for the light to reach the glass sheet, the spaceship moves up some distance, and, during the equal time for the light to continue to the far wall, the spaceship moves up a greater distance. So, to observers in the spaceship the light has followed a downward curving path (Figure 36.5). In this accelerating frame of reference, the light ray is deflected downward toward the floor, just as the thrown ball in Figure 36.4 is deflected. The curvature of the slow-moving ball is very pronounced; but if the ball were somehow thrown horizontally across the spaceship cabin at a velocity equal to that of light, its curvature would match the light ray's curvature.

Einstein actually imagined himself in elevators, certainly more common at the time than spaceships!

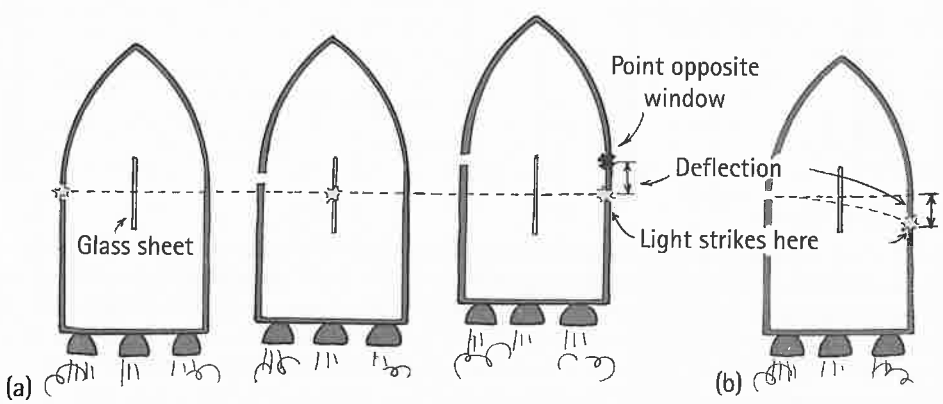


FIGURE 36.5

(a) An outside observer sees light travel horizontally in a straight line, and, like the ball in the previous figure, it strikes the wall slightly below a point opposite the window. (b) To an inside observer, the light bends as if responding to a gravitational field.

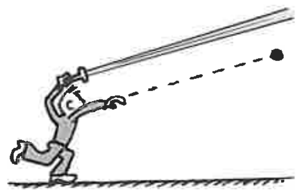


FIGURE 36.6

The trajectory of a flashlight beam is identical to the trajectory that a baseball would have if it could be "thrown" at the speed of light. Both paths curve equally in a uniform gravitational field.

An observer inside the ship feels "gravity" because of the ship's acceleration. The observer is not surprised by the deflection of the thrown ball, but might be quite surprised by the deflection of light. According to the principle of equivalence, if light is deflected by acceleration, it must be deflected by gravity. Yet how can gravity bend light? According to Newton's physics, gravitation is an interaction between masses; a moving ball curves because of the interaction between its mass and the mass of Earth. But what of light, which is pure energy and is massless? Einstein's answer was that light may be massless, but it's not "energyless." Gravity pulls on the energy of light because energy is equivalent to mass.

This was Einstein's first answer, before he fully developed the general theory of relativity. Later, he gave a deeper explanation—that light bends when it travels in a spacetime geometry that is bent. We shall see later in this chapter that the presence of mass results in the bending or warping of spacetime. The mass of Earth is too small to appreciably warp the surrounding spacetime, which is practically flat, so any such bending of light in our immediate environment is not ordinarily detected. Close to bodies of mass much greater than Earth's, however, the bending of light is large enough to detect.

Einstein predicted that starlight passing close to the Sun would be deflected by an angle of 1.75 seconds of arc—large enough to be measured. Although stars are not visible when the Sun is in the sky, the deflection of starlight can be observed during an eclipse of the Sun. (Measuring this deflection has become a standard practice at every total eclipse since the first measurements were made during the total eclipse of 1919.) A photograph of a darkened sky around the eclipsed Sun reveals the presence of the nearby bright stars. The positions of the stars are compared with those in other photographs of the same area taken at other times in the night with the same telescope. In every instance, the deflection of starlight has supported Einstein's prediction (Figure 36.7).

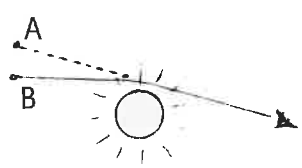


FIGURE 36.7

Starlight bends as it grazes the Sun. Point A shows the apparent position; point B shows the true position.

Light bends in Earth's gravitational field also—but not as much. We don't notice it because the effect is so tiny. For example, in a constant gravitational field of 1 g, a beam of horizontally directed light will "fall" a vertical distance of 4.9 m in 1 s (just as a baseball would), but it will travel a horizontal distance of 300,000 km in that time. Its curve would hardly be noticeable when you're this far from the beginning point. But if the light traveled 300,000 km in multiple reflections between idealized parallel mirrors, the effect would be quite noticeable (Figure 36.8). (Doing this would make a dandy home project for extra credit—like earning credit for a Ph.D.)

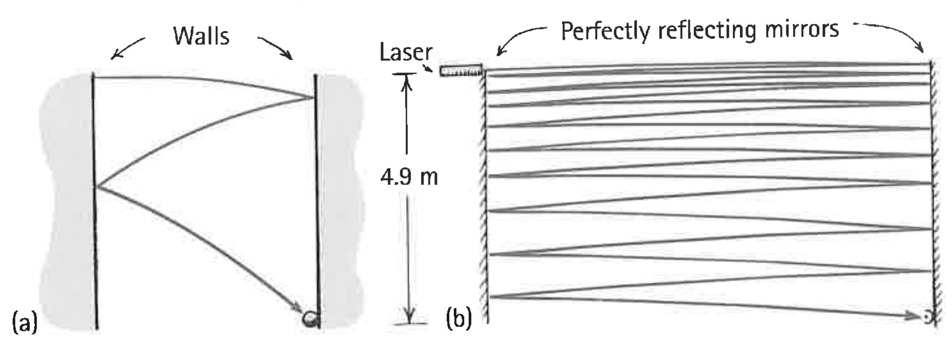


FIGURE 36.8

(a) If a ball is horizontally projected between a vertical pair of parallel walls, it will bounce back and forth and fall a vertical distance of 4.9 m in 1 s. (b) If a horizontal beam of light is directed between a vertical pair of perfectly parallel ideal mirrors, it will reflect back and forth and fall a vertical distance of 4.9 m in 1 s. The number of back-and-forth reflections is overly simplified in the diagram; if the mirrors were 300 km apart, for example, 1000 reflections would occur in 1 s.

CHECK POINT

1. Whoa! We learned previously that the pull of gravity is an interaction between masses. And we learned that light has no mass. Now we say that light can be bent by gravity. Isn't this a contradiction?
2. Why do we not notice the bending of light in our everyday environment?

Check Your Answers

1. There is no contradiction when the mass–energy equivalence is understood. It's true that light has no mass, but it is not "energyless." The fact that gravity deflects light is evidence that gravity pulls on the energy of light. Energy indeed is equivalent to mass!
2. Only because light travels so fast; just as, over a short distance, we do not notice the curved path of a high-speed bullet, we do not notice the curving of a light beam.

Gravity and Time: Gravitational Red Shift

Astrophysics goes beyond describing how the sky looks by explaining how it got to be as it is.

According to Einstein's general theory of relativity, gravitation causes time to slow down. If you move in the direction in which the gravitational force acts—from the top of a skyscraper to the ground floor, for instance, or from the surface of Earth to the bottom of a well—time will run slower at the point you reach than at the point you left behind. We can understand the slowing of clocks by gravity by applying the principle of equivalence and time dilation to an accelerating frame of reference.

Imagine our accelerating reference frame to be a large rotating disk. Suppose we measure time with three identical clocks, one placed on the disk at its center, a second placed on the rim of the disk, and the third at rest on the ground nearby (Figure 36.9). From the laws of special relativity, we know that the clock attached to the center, since it is not moving with respect to the ground, should run at the same rate as the clock on the ground—but not at the same rate as the clock attached to the rim of the disk. The clock at the rim is in motion with respect to the ground and should therefore be observed to be running more slowly than the ground clock—and therefore more slowly than the clock at the center of the disk. Although the clocks on the disk are attached to the same frame of reference, they do not run synchronously; the outer clock runs slower than the inner clock.

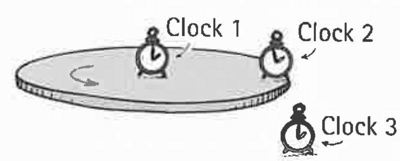


FIGURE 36.9

Clocks 1 and 2 are on an accelerating disk, and clock 3 is at rest in an inertial frame. Clocks 1 and 3 run at the same rate, while clock 2 runs slower. From the point of view of an observer at clock 3, clock 2 runs slow because it is moving. From the point of view of an observer at clock 1, clock 2 runs slow because it is at a lower potential (it would take work to move it from the edge to the center).

An observer on the rotating disk and an observer at rest on the ground both see the same difference in clock rates between themselves and the clock on the rim. Interpretations of the difference for the two observers are not the same, however. To the observer on the ground, the slower rate of the clock on the rim is due to its motion. But, to an observer on the rotating disk, the disk clocks are not in motion with respect to each other; instead, a centrifugal force acts on the clock at the rim, while no such force acts on the clock at the center. The observer on the disk is likely to conclude that the centrifugal force has something to do with the slowing of time. He notices that as he moves in the direction of the centrifugal force, outward from the center to the edge of the disk, time is slowed. By applying the principle of equivalence, which says that any effect of acceleration can be duplicated by gravity, we must conclude that as we move in the direction in which a gravitational force acts, time will also be slowed.

This slowing down will apply to all "clocks," whether physical, chemical, or biological. An executive working on the ground floor of a tall city skyscraper will age more slowly than her twin sister working on the top floor. The difference is very small, only a few millionths of a second per decade, because, by cosmic standards, the distance is small and the gravitation is weak. For larger differences in gravitation,

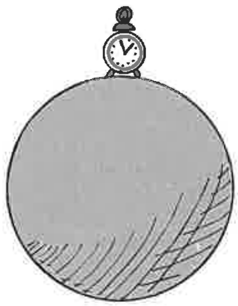


FIGURE 36.10

If you move from a distant point down to the surface of Earth, you move in the direction in which the gravitational force acts—toward a location where clocks run more slowly. A clock at the surface of Earth runs slower than a clock farther away.

such as between the surface of the Sun and the surface of Earth, the differences in time are larger (although still tiny). A clock at the surface of the Sun should run measurably slower than a clock at the surface of Earth. Years before he completed his general relativity theory, Einstein suggested a way to measure this when he formulated the principle of equivalence in 1907.

All atoms emit light at specific frequencies characteristic of the vibrational rate of electrons within the atom. Every atom is therefore a "clock," and a slowing down of atomic vibration indicates the slowing down of such clocks. An atom on the Sun should emit light of a lower frequency (slower vibration) than light emitted by the same element on Earth. Since red light is at the low-frequency end of the visible spectrum, a lowering of frequency shifts the color toward the red. This effect is called the **gravitational red shift.** The gravitational red shift is observed in light from the Sun, but various disturbing influences prevent accurate measurements of this tiny effect. It wasn't until 1960 that an entirely new technique, using gamma rays from radioactive atoms, permitted incredibly precise and confirming measurements of the gravitational slowing of time between the top and bottom floors of a laboratory building at Harvard University.[1]

So measurements of time depend not only on relative motion, as we learned in the last chapter, but also on gravity. In special relativity, time dilation depends on the *speed* of one frame of reference relative to another one. In general relativity, the gravitational red shift depends on the *location* of one point in a gravitational field relative to another one. As viewed from Earth, a clock will be measured to tick more slowly on the surface of a star than on Earth. If the star shrinks, its surface moves inward to ever-stronger gravity, which causes time on its surface to slow down more and more. We would measure longer intervals between the ticks of the star clock. But if we made our measurements of the star clock from the star itself, we would notice nothing unusual about the clock's ticking.

Suppose, for example, that an indestructible volunteer stands on the surface of a giant star that begins collapsing. We, as outside observers, will note a progressive slowing of time on the clock of our volunteer as the star surface recedes to regions of stronger gravity. The volunteer himself, however, does not notice any differences in his own time. He is viewing events within his own frame of reference, and he notices nothing unusual. As the collapsing star proceeds toward becoming a black hole and time proceeds normally from the viewpoint of the volunteer, we on the outside perceive time for the volunteer as approaching a complete stop; we see him frozen in time with an infinite duration between the ticks of his clock or the beats of his heart. From our view, his time stops completely. The gravitational red shift, instead of being a tiny effect, is dominating.

We can understand the gravitational red shift from another point of view—in terms of the gravitational force acting on photons. As a photon flies from the surface of a star, it is "retarded" by the star's gravity. It loses energy (but not speed). Since a photon's frequency is proportional to its energy, its frequency decreases as its energy decreases. When we observe the photon, we see that it has lower frequency than what it would if it had been emitted by a less-massive source. Its time has been slowed, just like the ticking of a clock is slowed. In the case of a black hole, a photon is unable to escape at all. It loses all its energy and all its frequency in the attempt. Its frequency is gravitationally red-shifted to zero, consistent with our observation that the rate at which time passes on a collapsing star approaches zero.

The Global Positioning System (GPS) must take account of the effect of gravity as well as speed on orbiting atomic clocks. Because of gravity, clocks run faster in orbit. Because of speed, they run slower. The effects vary during each elliptical orbit and they don't cancel. When your GPS unit tells you exactly where you are, thank Einstein.

[1]In the late 1950s, shortly after Einstein's death, the German physicist Rudolph Mössbauer discovered an important effect in nuclear physics that provides an extremely accurate method of using atomic nuclei as atomic clocks. The *Mössbauer effect,* for which its discoverer was awarded the Nobel Prize, has many practical applications. In late 1959, Robert Pound and Glen Rebka at Harvard University conceived an application that was a test for general relativity and performed the confirming experiment.

It is important to note the relativistic nature of time both in special relativity and in general relativity. In both theories, there is no way that you can extend the duration of your own existence. Others moving at different speeds or in different gravitational fields may attribute a great longevity to you, but your longevity is seen from *their* frame of reference, never from your own. Changes in time are always attributed to "the other guy."

CHECK POINT

Will a person at the top of a skyscraper age more than or less than a person at ground level?

Check Your Answer

More—going from the top of the skyscraper to the ground is going in the direction of the gravitational force, so it is going to a place where time runs more slowly.

Gravity and Space: Motion of Mercury

From the special theory of relativity, we know that measurements of space and time undergo transformations when motion is involved. Likewise with the general theory: Measurements of space differ in different gravitational fields—for example, close to and far away from the Sun.

Planets orbit the Sun and stars in elliptical orbits and move periodically into regions farther from the Sun and closer to the Sun. Einstein directed his attention to the varying gravitational fields experienced by the planets orbiting the Sun and found that the elliptical orbits of the planets should *precess* (Figure 36.11)—independently of the Newtonian influence of other planets. Near the Sun, where the effect of gravity on time is the greatest, the rate of precession should be the greatest; and far from the Sun, where time is less affected, any deviations from Newtonian mechanics should be virtually unnoticeable.

FIGURE 36.11
A precessing elliptical orbit.

Mercury, the planet nearest the Sun, is in the strongest part of the Sun's gravitational field. If the orbit of any planet exhibits a measurable precession, it should be Mercury, and the fact that the orbit of Mercury does precess—above and beyond effects attributable to the other planets—had been a mystery to astronomers since the early 1800s. Careful measurements showed that Mercury's orbit precesses about 574 seconds of arc per century. Perturbations by the other planets were found to account for all but 43 seconds of arc per century. Even after all known corrections due to possible perturbations by other planets had been applied, the calculations of physicists and astronomers failed to account for the extra 43 seconds of arc. Either Venus was extra massive or a never-discovered other planet (called Vulcan) was pulling on Mercury. And then came the explanation of Einstein, whose general relativity field equations applied to Mercury's orbit predict an extra 43 seconds of arc per century!

The mystery of Mercury's orbit was solved, and a new theory of gravity was recognized. Newton's law of gravitation, which had stood as an unshakable pillar of science for more than two centuries, was found to be a special limiting case of Einstein's more general theory. If the gravitational fields are comparatively weak, Newton's law turns out to be a good approximation of the new law—enough so that Newton's law, which is easier to work with mathematically, is the law that today's space scientists use most of the time.

Gravity, Space, and a New Geometry

We can begin to understand that measurements of space are altered in a gravitational field by again considering the accelerated frame of reference of our rotating disk. Suppose that we measure the circumference of the outer rim with a measuring stick. Recall the Lorentz contraction from special relativity: The measuring stick will appear contracted to any observer not moving along with the stick, while the dimensions of an identical measuring stick moving much more slowly near the center will be nearly unaffected (Figure 36.12). All distance measurements along a *radius* of the rotating disk should be completely unaffected by motion, because motion is perpendicular to the radius. Since only distance measurements parallel to and around the circumference are affected, the ratio of circumference to diameter when the disk is rotating is no longer the fixed constant π (3.14159 . . .) but is a variable depending on angular speed and the diameter of the disk.

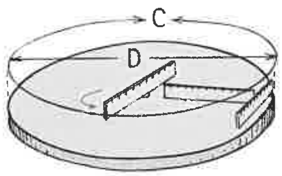


FIGURE 36.12

A measuring stick along the edge of the rotating disk appears contracted, while a measuring stick farther in and moving more slowly is not contracted as much. A measuring stick along a radius is not contracted at all. When the disk is not rotating, $C/D = \pi$; but, when the disk is rotating, C/D is not equal to π and Euclidean geometry is no longer valid. Likewise in a gravitational field.

According to the principle of equivalence, the rotating disk is equivalent to a stationary disk with a strong gravitational field near its edge and a progressively weaker gravitational field toward its center. Measurements of distance, then, will depend on the strength of gravitational field (or, more exactly, for relativity buffs, on gravitational potential), even if no relative motion is involved. Gravity causes space to be non-Euclidean; the laws of Euclidean geometry taught in high school are no longer valid when applied to objects in the presence of strong gravitational fields.

The standard model of cosmology assumes a flat universe dominated by dark matter and dark energy that formed by rapid inflation from its hot, dense origins.

The familiar rules of Euclidean geometry pertain to various figures you can draw on a flat surface. The ratio of the circumference of a circle to its diameter is equal to π; all the angles in a triangle add up to 180°; the shortest distance between two points is a straight line. The rules of Euclidean geometry are valid in flat space; but if you draw these figures on a curved surface, like a sphere or a saddle-shaped object, the Euclidean rules no longer hold (Figure 36.13). If you measure the sum of the angles for a triangle in space, you call the space flat if the sum is equal to 180°, spherelike or positively curved if the sum is larger than 180°, and saddlelike or negatively curved if it is less than 180°.

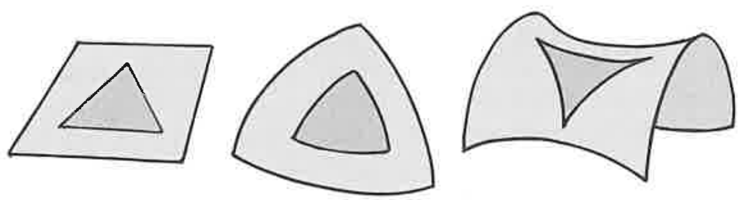


FIGURE 36.13

The sum of the angles of a triangle depends on which kind of surface the triangle is drawn on. (a) On a flat surface, the sum is 180°. (b) On a spherical surface, the sum is greater than 180°. (c) On a saddle-shaped surface, the sum is less than 180°.

Of course, the lines forming the triangles in Figure 36.13 are not all "straight" from a three-dimensional view, but they are the "straightest," or *shortest*, distances between two points if we are confined to the curved surface. These lines of shortest distance are called *geodesic* lines or simply **geodesics.**

The path of a light beam follows a geodesic. Suppose three experimenters on Earth, Venus, and Mars measure the angles of a triangle formed by light beams traveling between these three planets. The light beams bend when passing the Sun, resulting in the sum of the three angles being larger than 180° (Figure 36.14). So the space around the Sun is positively curved. The planets that orbit the Sun travel along four-dimensional geodesics in this positively curved spacetime. Freely falling objects, satellites, and light rays all travel along geodesics in four-dimensional spacetime.

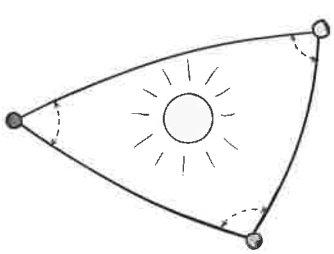


FIGURE 36.14

The light rays joining the three planets form a triangle. Since light passing near the Sun bends, the sum of the angles of the resulting triangle is greater than 180°.

"Small" parts of the universe are certainly curved. What about the universe as a whole? Recent study of the low-temperature radiation in space that is a remnant of the Big Bang suggests that the universe is flat. If it were open-ended like the saddle in Figure 36.13c, it would extend forever and beams of light that started out parallel would diverge. If it were closed like the spherical surface in Figure 36.13b, beams of light that started out parallel would eventually cross and circle back to their starting point. In such a universe, if you could look infinitely into space through an ideal telescope, you would see the back of your own head (after waiting patiently for enough billions of years)! In our actual flat universe, parallel beams of light remain parallel and will never return.

General relativity calls for a new geometry: Rather than space simply being a region of nothingness, space is a flexible medium that can bend and twist. How it bends and twists describes a gravitational field. General relativity is a geometry of curved, four-dimensional spacetime.[2] The mathematics of this geometry is too formidable to present here. The essence, however, is that the presence of mass produces the curvature, or *warping*, of spacetime. Conversely, a curvature of spacetime shows that mass must be present. Instead of visualizing gravitational forces between masses, we abandon altogether the notion of force and instead think of masses responding in their motion to the warping of the spacetime they inhabit. It is the bumps, depressions, and warpings of geometrical spacetime that *are* the phenomena of gravity.

We cannot visualize the four-dimensional bumps and depressions in spacetime because we are three-dimensional beings. We can get a glimpse of this warping by considering a simplified analogy in two dimensions: a heavy ball resting on the middle of a waterbed. The more massive the ball, the greater it dents or warps the two-dimensional surface. A marble rolled across the bed, but far from the ball, will roll in a relatively straight-line path, whereas a marble rolled near the ball will curve as it rolls across the indented surface. If the curve closes upon itself, its shape resembles an ellipse. The planets that orbit the Sun similarly travel along four-dimensional geodesics in the warped spacetime about the Sun.

Gravitational Waves

Every object has mass and therefore warps the surrounding spacetime. When an object undergoes a change in motion, the surrounding warp moves in order to readjust to the new position. These readjustments produce ripples in the overall geometry of spacetime. This is similar to moving a ball that rests on the surface of a waterbed. A disturbance ripples across the waterbed surface in waves; if we move a more massive ball, then we get a greater disturbance and the production of even stronger waves. Similarly for spacetime in the universe. Similar ripples travel outward from a gravitational source at the speed of light and are **gravitational waves.**

Any accelerating object produces a gravitational wave. In general, the more massive the object and the greater its acceleration, the stronger the resulting gravitational wave. But even the strongest waves produced by ordinary astronomical events are extremely weak—the weakest known in nature. For example, the gravitational waves emitted by a vibrating electric charge are a trillion trillion trillion times weaker than the electromagnetic waves emitted by the same charge. Detecting gravitational waves is enormously difficult, and no confirmed detection has occurred to

[2]Don't be discouraged if you cannot visualize four-dimensional spacetime. Einstein himself often told his friends, "Don't try. I can't do it either." Perhaps we are not too different from the great thinkers of Galileo's time who couldn't think of a moving Earth!

fyi

- One of general relativity's predictions is a subtle twisting of spacetime around a massive spinning object. A test for this "frame-dragging" effect would be predictable tiny changes in the orientations of satellite orbits and orbiting gyroscopes. Researchers in 2004 found such confirming evidence.

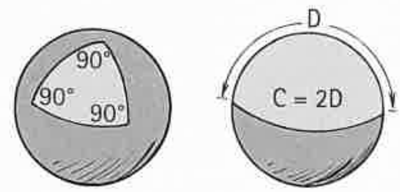


FIGURE 36.15

The geometry of the curved surface of Earth differs from the Euclidean geometry of flat space. Note, in the globe on the left, that the sum of the angles for an equilateral triangle in which each side equals 1/4 Earth's circumference is clearly greater than 180°. The globe on the right shows Earth's circumference is only twice its diameter instead of 3.14 times its diameter. Euclidean geometry is also invalid in curved space.

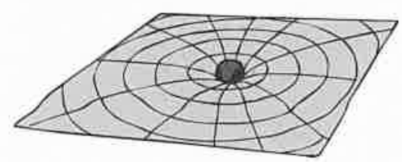


FIGURE 36.16

A two-dimensional analogy of four-dimensional warped spacetime. Spacetime near a star is curved in a way similar to the surface of a waterbed when a heavy ball rests on it.

fyi

- Researchers in 2005 confirmed predictions of energy losses due to gravitational waves emitted by the double-pulsar system PSR J0737-3039A/B over a 3-year period. Yay, Einstein!

Space is stretching out, carrying the galaxies with it. Visible light in the early universe has stretched out now to be relatively long-wavelength microwave radiation.

date. Recently completed wave detectors are expected to detect gravitational waves from supernovae, which may radiate away as much as 0.1% of their mass as gravitational waves, and perhaps from even more cataclysmic events, such as colliding black holes.

As weak as they are, gravitational waves are everywhere. Shake your hand back and forth—you have just produced a gravitational wave. It is not very strong, but it exists.

Newtonian and Einsteinian Gravitation

When Einstein formulated his new theory of gravitation, he realized that if his theory is valid, his field equations must reduce to Newtonian equations for gravitation in the weak-field limit. He showed that Newton's law of gravitation is a special case of the broader theory of relativity. Newton's law of gravitation is still an accurate description of most of the interactions between bodies in the solar system and beyond. From Newton's law, one can calculate the orbits of comets and asteroids and even predict the existence of undiscovered planets. Even today, when computing the trajectories of space probes to the Moon and planets, only ordinary Newtonian theory is used. This is because the gravitational field of these bodies is very weak, and, from the viewpoint of general relativity, the surrounding spacetime is essentially flat. But for regions of more intense gravitation, where spacetime is more appreciably curved, Newtonian theory cannot adequately account for various phenomena—such as the precession of Mercury's orbit close to the Sun and, in the case of stronger fields, the gravitational red shift and other apparent distortions in measurements of space and time. These distortions reach their limit in the case of a star that collapses to a black hole, where spacetime completely folds over on itself. Only Einsteinian gravitation reaches into this domain.

We saw, in Chapter 32, that Newtonian physics is linked at one end with quantum theory, whose domain is the very light and very small—tiny particles and atoms. And now we have seen that Newtonian physics is linked at the other end with relativity theory, whose domain is the very massive and very large. We do not see the world the way the ancient Egyptians, Greeks, or Chinese did. It is unlikely that people in the future will see the universe as we do. Our view of the universe may be quite limited, and perhaps filled with misconceptions, but it is most likely clearer than the views of others before us. Our view today stems from the findings of Copernicus, Galileo, Newton, and, more recently, Einstein—findings that were often opposed on the grounds that they diminished the importance of humans in the universe. In the past, being important meant having risen above nature—being apart from nature. We have expanded our vision since then by enormous effort, painstaking observation, and an unrelenting desire to comprehend our surroundings. Seen from today's understanding of the universe, we find our importance in being very much a part of nature, not apart from it. We are the part of nature that is becoming more and more conscious of itself.

If a learner's first course in physics is enjoyable, the rigor of a second course will be welcome and meaningful.

SUMMARY OF TERMS

General theory of relativity The second of Einstein's theories of relativity, which relates gravity to the properties of space and time.

Principle of equivalence Because observations made in an accelerated frame of reference are indistinguishable from observations made in a gravitational field, any effect produced by gravity can be duplicated by accelerating a frame of reference.

Gravitational red shift The lengthening of the waves of electromagnetic radiation escaping from a massive object.

Geodesic The shortest path between two points in various models of space.

Gravitational wave A gravitational disturbance, generated by an accelerating mass, that propagates through spacetime.

REVIEW QUESTIONS

Reference Frames—Nonaccelerated and Accelerated

1. What is the principal difference between the theory of special relativity and the theory of general relativity?

Principle of Equivalence

2. In a spaceship accelerating at *g*, far from Earth's gravity, how does the motion of a dropped ball compare with the motion of a ball dropped at Earth's surface?
3. Exactly what is *equivalent* in the principle of equivalence?

Bending of Light by Gravity

4. Compare the bending of the paths of baseballs and photons by a gravitational field.
5. Why must the Sun be eclipsed to measure the deflection of starlight passing near the Sun?

Gravity and Time: Gravitational Red Shift

6. What is the effect of strong gravitation on measurements of time?
7. Which runs slower, a clock at the top of the tallest skyscraper in Chicago or a clock on the shore of Lake Michigan?
8. How does the frequency of a particular spectral line observed in sunlight compare with the frequency of that line observed from a source on Earth?
9. If we view events occurring on a star that is collapsing to become a black hole, do we see time speeded up or slowed down?

Gravity and Space: Motion of Mercury

10. Of all the planets, why is Mercury the best candidate for finding evidence of the relationship between gravitation and space?
11. In what kind of gravitational field are Newton's laws valid?

Gravity, Space, and a New Geometry

12. A measuring stick placed along the circumference of a rotating disk will appear contracted, but if it is oriented along a radius, it will not. Explain.
13. The ratio of circumference to diameter for measured circles on a disk equals π when the disk is at rest, but not when the disk is rotating. Explain.
14. What effect does mass have on spacetime?

Gravitational Waves

15. What occurs in the surrounding space when a massive object undergoes a change in its motion?
16. A star 10 light-years away explodes and produces gravitational waves. How long will it take these waves to reach Earth?
17. Why are gravitational waves so difficult to detect?

Newtonian and Einsteinian Gravitation

18. Does Einstein's theory of gravitation invalidate Newton's theory of gravitation? Explain.
19. Is Newtonian physics adequate to get a rocket to the Moon?
20. How does Newtonian physics link with quantum theory and relativity theory?

EXERCISES

1. What is different about the reference frames that apply to special relativity and to general relativity?
2. An astronaut awakes in her closed capsule, which actually sits on the Moon. Can she tell whether her weight is the result of gravitation or of accelerated motion? Explain.
3. Provide a classical explanation for an astronaut in an orbiting spacecraft experiencing no net force (as measured by a weighing scale), even though the astronaut is in the grips of Earth gravity.
4. An astronaut is provided a "gravity" when the spaceship's engines are activated to accelerate the ship. This requires the use of fuel. Is there a way to accelerate and provide "gravity" without the sustained use of fuel? Explain, perhaps using ideas from Chapter 8.
5. In a spaceship far from the reaches of gravity, under what conditions could you feel as if the spaceship were stationary on Earth's surface?
6. In his famous novel *Journey to the Moon,* Jules Verne stated that occupants in a spaceship would shift their orientation from up to down when the ship crossed the point where the Moon's gravitation became greater than Earth's. Is this correct? Defend your answer.
7. What happens to the separation distance between two people if they both walk north at the same rate from two locations on Earth's equator? And just for fun, where in the world is a step in every direction a step south?
8. We readily note the bending of light by reflection and refraction, but why are we not aware of the bending of light by gravity?
9. Light *does* bend in a gravitational field. Why is this bending not taken into consideration by surveyors who use laser beams as straight lines?
10. Why do we say that light travels in straight lines? Is it strictly accurate to say that a laser beam provides a perfectly straight line for purposes of surveying? Explain.
11. Your friend says that light passing the Sun is bent whether or not Earth experiences a solar eclipse. Do you agree or disagree, and why?
12. In 2004, when Mercury passed between the Sun and Earth, sunlight was not appreciably bent as it passed Mercury. Why?
13. A setting Sun is seen as distorted on Earth, but not by astronauts on the Moon. What causes this distortion (and why could this question have been asked back in Chapter 28)?

14. At the end of 1 s, a horizontally fired bullet drops a vertical distance of 4.9 m from its otherwise straight-line path in a gravitational field of 1 *g*. By what distance would a beam of light drop from its otherwise straight-line path if it traveled in a uniform field of 1 *g* for 1 s? For 2 s?
15. Light changes its energy when it "falls" in a gravitational field. This change in energy is not evidenced by a change in speed, however. What is the evidence for this change in energy?
16. Would we notice a slowing down or speeding up of a clock if we carried it to the bottom of a very deep well?
17. If we witness events occurring on the Moon, where gravitation is weaker than on Earth, would we expect to see a gravitational red shift or a gravitational blue shift? Explain.
18. Armed with highly sensitive detection equipment, you are in the front of a railroad car that is accelerating forward. Your friend at the rear of the car shines green light toward you. Do you find the light to be red-shifted (lowered in frequency), blue-shifted (increased in frequency), or neither? Explain. (*Hint:* Think in terms of the principle of equivalence. What is your accelerating railroad car equivalent to?)
19. Why will the gravitational field intensity increase on the surface of a shrinking star?
20. Will a clock at the equator run slightly faster or slightly slower than an identical clock at one of Earth's poles?
21. Do you age faster at the top of mountain or at sea level?
22. Splitting hairs, should a person who worries about growing old live at the top or at the bottom of a tall apartment building?
23. Which would run slower, a clock at the center of a rotating space habitat or one at the edge? Or would there be no difference?
24. Prudence and Charity are twins raised at the center of a rotating kingdom. Charity goes to live at the edge of the kingdom for a time and then returns home. Which twin is older when they rejoin? (Ignore any time-dilation effects associated with travel to and from the edge.)
25. Splitting hairs, if you shine a beam of colored light to a friend above in a high tower, will the color of light your friend receives be the same color you send? Explain.
26. Is light emitted from the surface of a massive star red-shifted or blue-shifted by gravity?
27. From our frame of reference on Earth, objects slow to a stop as they approach black holes in space because time gets infinitely stretched by the strong gravity near the black hole. If astronauts accidentally falling into a black hole tried to signal back to Earth by flashing a light, what kind of "telescope" would we need to detect the signals?
28. Would an astronaut falling into a black hole see the outside universe red-shifted or blue-shifted?
29. How can we "observe" a black hole if neither matter nor radiation can escape from it?
30. Should it be possible in principle for a photon to circle a very massive star?
31. Why does the gravitational attraction between the Sun and Mercury vary? Would it vary if the orbit of Mercury were perfectly circular?
32. Your friend whimsically says that at the North Pole, a step in any direction is a step south. Do you agree?
33. In the astronomical triangle shown in Figure 36.14, with sides defined by light paths, the sum of the interior angles is more than 180°. Is there any astronomical triangle whose interior angles sum to less than 180°?
34. Do binary stars (double-star systems that orbit about a common center of mass) radiate gravitational waves? Why or why not?
35. Given the possible sources of gravitational waves in the universe, would you expect them to have short wavelengths or long?
36. Based on what you know about the emission and absorption of electromagnetic waves, suggest how gravitational waves are emitted and how they are absorbed. (Scientists seeking to detect gravitational waves must arrange for them to be absorbed.)
37. Comparing Einstein's and Newton's theories of gravitation, how can the correspondence principle be applied?
38. Current findings suggest that the universe is flat. What is an implication of this finding?
39. Make up a multiple-choice question to check a classmate's understanding of the principle of equivalence.
40. Make up a multiple-choice question to check a classmate's understanding of the effect of gravity on time.

CHAPTER 36 ONLINE RESOURCES

PhysicsPlace.com™

Quizzes

Flashcards

Links

PART EIGHT MULTIPLE-CHOICE PRACTICE EXAM

Choose the BEST answer to the following:

1. What Einstein discovered about space and time is that they
   (a) are separate entities.
   (b) are parts of one whole.
   (c) follow an inverse-square law.
   (d) are special to space travelers.
2. In his special theory of relativity, Einstein stated that the laws of physics are
   (a) different in different situations.
   (b) common sense applied to microscopic and macroscopic things.
   (c) the same in all frames of reference.
   (d) the same in all uniformly moving frames of reference.
3. Einstein's second postulate tells us that the speed of light
   (a) depends on one's frame of reference.
   (b) is a constant in all frames of reference.
   (c) provides accurate clocks.
   (d) slows in a transparent medium.
4. When we speak of time dilation, we mean that time
   (a) compresses with speed.
   (b) stretches with speed.
   (c) is a constant at all speeds.
   (d) is related to space.
5. If you travel at high speed, then compared with your friends who "stay at home," you are
   (a) older.
   (b) younger.
   (c) no younger nor no older.
   (d) longer.
6. Clocks on a fast-moving spaceship whizzing past Earth appear to run slow when viewed from
   (a) inside the spaceship.
   (b) Earth.
   (c) Both of these.
   (d) Neither of these.
7. If you were to travel at a speed close to that of the speed of light, you could notice that your own
   (a) mass changes.
   (b) pulse decreases.
   (c) Both of these.
   (d) Neither of these.
8. As a blinking light source approaching you gains in speed, you see the frequency of flashes
   (a) increase.
   (b) decrease.
   (c) remain unchanged.
   (d) None of these.
9. At very high speeds, an object appears to an observer at rest to be
   (a) shorter in the direction of travel.
   (b) shrunken in all directions.
   (c) shorter in the direction perpendicular to travel.
   (d) longer in all directions.
10. Compared with the Newtonian momentum $p = mv$, the momentum of an object traveling at great speed is
    (a) greater.
    (b) less.
    (c) the same.
    (d) dependent on rest mass.
11. Relativity equations for time, length, and momentum hold true for
    (a) everyday low speeds.
    (b) relativistic speeds.
    (c) Both of these.
    (d) Neither of these.
12. To say that $E = mc^2$ is to say that energy
    (a) increases as the speed of light squared.
    (b) is twice as much as the speed of light.
    (c) and mass are equivalent.
    (d) equals mass traveling at the speed of light squared.
13. According to the correspondence principle,
    (a) new theory must agree with old theory where they overlap.
    (b) Newton's mechanics is as valid as Einstein's mechanics.
    (c) relativity equations apply to high speeds while Newton's equations apply to low speeds.
    (d) special relativity and general relativity are two sides of the same coin.
14. Things that are equivalent according to the equivalence principle are
    (a) space and time.
    (b) a traveling twin and a stay-at-home twin.
    (c) gravity and acceleration.
    (d) mass and energy.
15. According to general relativity,
    (a) mass distorts spacetime.
    (b) gravity affects clocks.
    (c) light can't escape from a black hole.
    (d) All of these.
16. According to four-dimensional geometry, the angles of a triangle add to 180°
    (a) always.
    (b) sometimes.
    (c) never.
    (d) on planet Earth only.
17. General relativity predicts that
    (a) light leaving the Sun is slowed by gravity.
    (b) light passing the Sun is deflected.
    (c) a clock on the Sun's surface runs faster than on Earth.
    (d) All of these.
18. If a star that is 20 light-years from Earth explodes, gravitational waves from the explosion would reach Earth in
    (a) less than 20 years.
    (b) 20 years.
    (c) more than 20 years.
    (d) None of these.

After you have made thoughtful choices, and discussed them with your friends, find the answers on page 681.

EPILOGUE

I hope you've enjoyed *Conceptual Physics* and will value your knowledge of physics as a worthwhile component of your general education. Viewing physics as a study of the rules of nature can contribute to your sense of wonder and enhance the way you see the physical world—knowing that so much in nature is interconnected, with seemingly diverse phenomena often following the same basic rules. How intriguing that the rules governing a falling apple also apply to a space station orbiting Earth, that a sky's redness at sunset is connected to its blueness at midday, or that the link between electricity and magnetism reveals the nature of light.

The value of science is more than its applications to fast cars, iPods, computers, and other products of technology. Its greatest value lies in its methods of understanding and investigating nature—that hypotheses are framed so that they are capable of being disproved and experiments are designed so that their results can be reproduced by others. Science is more than a body of knowledge. It is a way of thinking.

But even now many, if not most, people hardly give a thought to science. Too many people around the world believe that all relevant knowledge is to be found in one or another sacred book. To such people, the answers to any questions of nature are to be found not in experimentation, but in the established written word. Once answers are found in the sacred text, there is no further investigation. Where there is no investigation, there is no science. Science becomes moot. Questioning and openness to new ideas stop.

This kind of thinking can get in the way of solving environmental problems, feeding ever more people, and finding cures for disease and other forms of human misery. Certitude ruled in the Dark Ages, and it may rule again. The beginning pages of this book cited the burning down of the Great Library of Alexandria by people who presumably were certain in their world view, people threatened by the heretical writing in the Great Library. Could it happen again—on a larger scale?

It is hard to imagine an end to the human adventure, but an end is painfully possible. This adventure started eons ago with specks of living matter making their appearance on this tiny planet orbiting an undistinguished star among countless other stars in one of billions of galaxies. Some of the specks replicated themselves, and the replicas replicated themselves ad infinitum—but with variations appearing in all the niches of ever-changing environments. Four hundred million years ago, before mammals, there were fish. Then came amphibians and then reptiles. In the struggle of species survival, trillions upon trillions of life forms passed their genetic traits on to their offspring, sometimes here and there making adaptive changes. After a long and prodigious ascent, some 200,000 years ago, humans emerged. We should not ignore the sacrifices of the innumerable lives that brought us to where we are. We should celebrate this long and astounding journey of life—for we are the benefactors. In my view, just as science is more fascinating than science fiction, the modern story of the ascent of humans is more beautiful than the legends written in sacred books.

Science offers a means of establishing our origins and shaping our future—at a time when the potential for world calamity has never been greater. Denial of overpopulation (the elephant in the room), indifference of the powerful to human suffering, energy greed, and forms of fanaticism devoted to terror, even nuclear terror, beset our age. Science offers the *tools* to save ourselves. The *will* is ours to find.

Fortunately, more and more people are finding the will and employing scientific tools to address the societal issues that threaten our survival. Growing numbers of imaginative, knowledgeable, caring people are focusing their intellectual and emotional energies to help solve global problems. Awareness of the urgency is growing. Earth is the home we all share. Science is needed to protect and preserve this home and to care for its billions of human occupants.

Appendix A

ON MEASUREMENT AND UNIT CONVERSIONS

Two major systems of measurement prevail in the world today: the *United States Customary System* (USCS, formerly called the British system of units), used in the United States of America and formerly in Burma, and the *Système International* (SI) (known also as the international system and as the metric system), used everywhere else. Each system has its own standards of length, mass, and time. The units of length, mass, and time are sometimes called the *fundamental units* because, once they are selected, other quantities can be measured in terms of them.

United States Customary System

Based on the British Imperial System, the USCS is familiar to everyone in the United States. It uses the foot as the unit of length, the pound as the unit of weight or force, and the second as the unit of time. The USCS is presently being replaced by the international system—rapidly in science and technology and some sports (track and swimming), but so slowly in other areas and in some specialties it seems the change may never come. For example, we will continue to buy seats on the 50-yard line. We remember camera film in millimeters and compact discs in inches.

For measuring time, there is no difference between the two systems except that in pure SI the only unit is the second (s, not sec) with prefixes; but in general, minute, hour, day, year, and so on, with two or more lettered abbreviations (h, not hr), are accepted in the USCS.

Systéme International

During the 1960 International Conference on Weights and Measures held in Paris, the SI units were defined and given status. Table A.1 shows SI units and their symbols. SI is based on the *metric system*, originated by French scientists after the French Revolution in 1791. The orderliness of this system makes it useful for scientific work, and it is used by scientists all over the world. The metric system branches into two systems of units. In one of these the unit of length is the meter, the unit of mass is the kilogram, and the unit of time is the second. This is called the *meter-kilogram-second* (mks) system and is preferred in physics. The other branch is the *centimeter-gram-second* (cgs) system, which, because of its smaller values, is favored in chemistry. The cgs and mks units are related to each other as follows: 100 centimeters equal 1 meter; 1000 grams equal 1 kilogram. Table A.2 shows several units of length related to each other.

One major advantage of a metric system is that it uses the decimal system, in which all units are related to smaller or larger units by dividing or multiplying by 10. The prefixes shown in Table A.3 are commonly used to show the relationship among units.

TABLE A.1

SI Units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Force	newton	N
Energy	joule	J
Current	ampere	A
Temperature	kelvin	K

TABLE A.2

Conversions Between Different Units of Length

Unit of Length	Kilometer	Meter	Centimeter	Inch	Foot	Mile
1 kilometer	= 1	1000	100,000	39,370	3280.84	0.62140
1 meter	= 0.00100	1	100	39.370	3.28084	6.21×10^{-4}
1 centimeter	= 1.0×10^{-5}	0.0100	1	0.39370	0.032808	6.21×10^{-6}
1 inch	= 2.54×10^{-5}	0.02540	2.5400	1	0.08333	1.58×10^{-5}
1 foot	= 3.05×10^{-4}	0.30480	30.480	12	1	1.89×10^{-4}
1 mile	= 1.60934	1609.34	160,934	63,360	5280	1

Appendix B

MORE ABOUT MOTION

When we describe the motion of something, we say how it moves relative to something else (Chapter 3). In other words, motion requires a reference frame (an observer, origin, and axes). We are free to choose this frame's location and to have it moving relative to another frame. When our frame of motion has zero acceleration, it is called an *inertial frame.* In an inertial frame, force causes an object to accelerate in accord with Newton's laws. When our frame of reference is accelerated, we observe fictitious forces and motions (Chapter 8). Observations from a carousel, for example, are different when it is rotating and when it is at rest. Our description of motion and force depends on our "point of view."

We distinguish between *speed* and *velocity* (Chapter 3). Speed is how fast something moves, or the time rate of change of position (excluding direction): a *scalar* quantity. Velocity includes direction of motion: a *vector* quantity whose magnitude is speed. Objects moving at constant velocity move the same distance in the same time in the same direction.

Another distinction between speed and velocity has to do with the difference between distance and net distance, or *displacement.* Speed is *distance per duration,* while velocity is *displacement per duration.* Displacement differs from distance. For example, a commuter who travels 10 kilometers to work and back travels 20 kilometers, but has "gone" nowhere. The distance traveled is 20 kilometers and the displacement is zero. Although the instantaneous speed and instantaneous velocity have the same value at the same instant, the average speed and average velocity can be very different. The average speed of this commuter's round trip is 20 kilometers divided by the total commute time—a value greater than zero. But the average velocity is zero. In science, displacement is often more important than distance. (To avoid information overload, we have not treated this distinction in the text.)

Acceleration is the rate at which velocity changes. This can be a change in speed only, a change in direction only, or both. Slowing down is often called *deceleration.*

In Newtonian space and time, space has three dimensions—length, width, and height—each with two directions. We can go, stop, and return in any of them. Time has one dimension, with two directions—past and future. We cannot stop or return, only go. In Einsteinian spacetime, these four dimensions merge (Chapter 35).

Computing Velocity and Distance Traveled on an Inclined Plane

Recall, from Chapter 2, Galileo's experiments with inclined planes. Consider a plane tilted such that the speed of a rolling ball increases at the rate of 2 meters per second each second—an acceleration of 2 m/s^2. At the instant it starts moving, its velocity is zero; and 1 second later, it is rolling at 2 m/s; at the end of the next second, 4 m/s; at the end of the next second, 6 m/s; and so on. Starting from rest, the velocity of the ball at any instant is simply

$$\text{Velocity} = \text{acceleration} \times \text{time}$$

or, in shorthand notation,

$$v = at$$

(It is customary to omit the multiplication sign, $\times$, when expressing relationships in mathematical form. When two symbols are written together, such as the *at* in this case, it is understood that they are multiplied.)

How fast the ball rolls is one thing; how *far* it rolls is another. To understand the relationship between acceleration and distance traveled, we must first investigate the relationship between instantaneous velocity and *average velocity.* If the ball shown in Figure B.1 starts from rest, it will roll a distance of 1 meter in the first second. Question: What will be its average speed? The answer is 1 m/s (because it covered 1 meter in the interval of 1 second). But we have seen that the *instantaneous velocity* at the end of the first second is 2 m/s. Since the acceleration is uniform, the average in any time interval is found the same way we usually find the average of any two numbers: add them and divide by 2. (Be careful not to do this when acceleration is not uniform!) So, if we add the initial speed (zero in this case) and the final speed of 2 m/s and then divide by 2, we get 1 m/s for the average velocity.

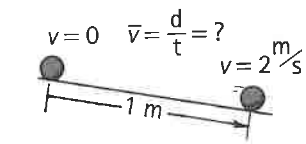


FIGURE B.1
The ball rolls 1 m down the incline in 1 s and reaches a speed of 2 m/s. Its average speed, however, is 1 m/s. Do you see why?

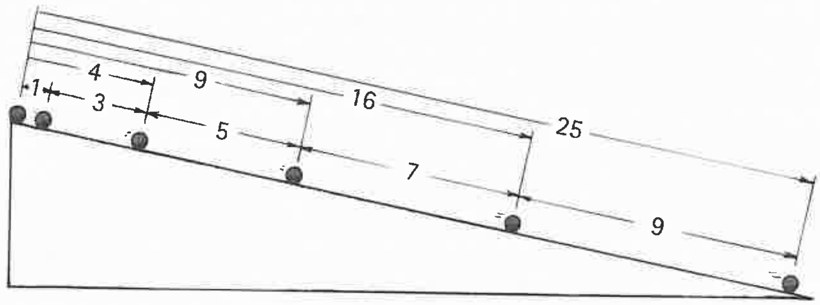


FIGURE B.2

If the ball covers 1 m during its first second, then, in each successive second, it will cover the odd-numbered sequence of 3, 5, 7, 9 m, and so on. Note that the total distance covered increases as the square of the total time.

In each succeeding second, we see the ball roll a longer distance down the same slope in Figure B.2. Note the distance covered in the second time interval is 3 meters. This is because the average speed of the ball in this interval is 3 m/s. In the next 1-second interval, the average speed is 5 m/s, so the distance covered is 5 meters. It is interesting to see that successive increments of distance increase as a *sequence of odd numbers.* Nature clearly follows mathematical rules!

CHECK POINT

During the span of the second time interval, the ball begins at 2 m/s and ends at 4 m/s. What is the *average speed* of the ball during this 1-s interval? What is its *acceleration?*

Check Your Answers

$$\text{Average speed} = \frac{\text{beginning} + \text{final speed}}{2} = \frac{2\text{ m/s} + 4\text{ m/s}}{2} = 3\text{ m/s}$$

$$\text{Acceleraion} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{4\text{ m/s} - 2\text{ m/s}}{1\text{ s}} = \frac{2\text{ m/s}}{1\text{ s}} = 2\text{ m/s}^2$$

Investigate Figure B.2 carefully and note the *total* distance covered as the ball accelerates down the plane. The distances go from zero to 1 meter in 1 second, zero to 4 meters in 2 seconds, zero to 9 meters in 3 seconds, zero to 16 meters in 4 seconds, and so on in succeeding seconds. The sequence for *total distances* covered is of the *squares of the time.* We'll investigate the relationship between distance traveled and the square of the time for constant acceleration more closely in the case of free fall.

Computing Distance When Acceleration Is Constant

How far will an object released from rest fall in a given time? To answer this question, let us consider the case in which it falls freely for 3 seconds, starting at rest. Neglecting air resistance, the object will have a constant acceleration of about 10 meters per second each second (actually more like 9.8 m/s^2, but we want to make the numbers easier to follow).

$$\begin{aligned}
\text{Velocity at the } \textit{beginning} &= 0\text{ m/s}\\
\text{Velocity at the } \textit{end} \text{ of 3 seconds} &= (10 \times 3)\text{ m/s}\\
\textit{Average} \text{ velocity} &= \tfrac{1}{2}\text{ the sum of these two speeds}\\
&= \tfrac{1}{2} \times (0 + 10 \times 3)\text{ m/s}\\
&= \tfrac{1}{2} \times 10 \times 3 = 15\text{ m/s}\\
\text{Distance traveled} &= \text{average velocity} \times \text{time}\\
&= (\tfrac{1}{2} \times 10 \times 3) \times 3\\
&= \tfrac{1}{2} \times 10 \times 3^2 = 45\text{ m}
\end{aligned}$$

We can see from the meanings of these numbers that

$$\text{Distance traveled} = \tfrac{1}{2} \times \text{acceleration} \times \text{square of time}$$

This equation is true for an object falling not only for 3 seconds but for any length of time, as long as the acceleration is constant. If we let d stand for the distance traveled, a for the acceleration, and t for the time, the rule may be written, in shorthand notation,

$$d = \tfrac{1}{2}at^2$$

This relationship was first deduced by Galileo. He reasoned that, if an object falls for, say, twice the time, it will fall with *twice the average speed.* Since it falls for *twice* the time at *twice* the average speed, it will fall *four* times as far. Similarly, if an object falls for *three* times the time, it will have an average speed *three* times as great and will fall *nine* times as far. Galileo reasoned that the total distance fallen should be proportional to the *square* of the time.

In the case of objects in free fall, it is customary to use the letter g to represent the acceleration instead of the letter a (g because acceleration is due to *gravity*). While the value of g varies slightly in different parts of the world, it is approximately equal to 9.8 m/s^2 (32 ft/s^2). If we use g for the acceleration of a freely falling object (negligible air resistance), the equations for falling objects starting from a rest position become

$$v = gt$$
$$d = \tfrac{1}{2}gt^2$$

Much of the difficulty in learning physics, like learning any discipline, has to do with learning the language—the many terms and definitions. Speed is somewhat different from velocity, and acceleration is vastly different from speed or velocity.

CHECK POINT

1. An auto starting from rest has a constant acceleration of $4\ m/s^2$. How far will it go in 5 s?
2. How far will an object released from rest fall in 1 s? In this case the acceleration is $g = 9.8\ m/s^2$.
3. If it takes 4 s for an object to freely fall to the water when released from the Golden Gate Bridge, how high is the bridge?

Check Your Answers

1. Distance $= \frac{1}{2} \times 4 \times 5^2 = 50$ m
2. Distance $= \frac{1}{2} \times 9.8 \times 1^2 = 4.9$ m
3. Distance $= \frac{1}{2} \times 9.8 \times 4^2 = 78.4$ m

Notice that the units of measurement when multiplied give the proper units of meters for distance:

$$d = \frac{1}{2} \times 9.8\ m/\cancel{s}^2 \times 16\cancel{s}^2 = 78.4\ m$$

Mass and weight are related but are different from each other. Likewise for work, heat, and temperature. Please be patient with yourself as you find that learning the similarities and the differences among physics concepts is not an easy task.

So far our equations for speed and distance have been for starting-from-rest cases. What of objects that undergo uniform acceleration and don't start from rest? A little thought will show that

$$v = v_0 + at$$

$$d = v_0 t + 1/2\, at^2$$

We've simply tacked on the initial conditions: velocity beginning with v_0 and distance traveled increased by $v_0 t$. Common sense tells you that when acceleration is zero, these equations become

$$v = v_0$$

$$d = v_0 t$$

After all, physics is applied common sense!

Appendix C

GRAPHING

Graphs: A Way to Express Quantitative Relationships

Graphs, like equations and tables, show how two or more quantities relate to each other. Since investigating relationships between quantities makes up much of the work of physics, equations, tables, and graphs are important physics tools.

Equations are the most concise way to describe quantitative relationships. For example, consider the equation $v = v_0 + gt$. It compactly describes how a freely falling object's velocity depends on its initial velocity, acceleration due to gravity, and time. Equations are nice shorthand expressions for relationships among quantities.

Tables give values of variables in list form. The dependence of v on t in $v = v_0 + gt$ can be shown by a table that lists various values v for corresponding times t. Table 3.2 on page 41 is an example. Tables are especially useful when the mathematical relationship between quantities is not known, or when numerical values must be given to a high degree of accuracy. Also, tables are handy for recording experimental data.

Graphs *visually* represent relationships between quantities. By looking at the shape of a graph, you can quickly tell a lot about how the variables are related. For this reason, graphs can help clarify the meaning of an equation or table of numbers. And, when the equation is not already known, a graph can help reveal the relationship between variables. Experimental data are often graphed for this reason.

Graphs are helpful in another way. If a graph contains enough plotted points, it can be used to estimate values between the points (interpolation) or values following the points (extrapolation).

Cartesian Graphs

The most common and useful graph in science is the *Cartesian* graph. On a Cartesian graph, possible values of one variable are represented on the vertical axis (called the *y-axis*) and possible values of the other variable are plotted on the horizontal axis (*x-axis*).

Figure C.1 shows a graph of two variables, x and y, that are *directly proportional* to each other. A direct proportionality is a type of *linear* relationship. Linear relationships have straight-line graphs—the easiest kinds of graphs to interpret. On the graph shown in Figure C.1, the continuous straight-line rise from left to right tells you that, as x increases, y increases. More specifically, it shows that y increases at a constant rate with respect to x. As x increases, y increases. The graph of a direct proportionality often passes through the "origin"—the point at the lower left where $x = 0$ and $y = 0$. In Figure C.1, however, we see the graph begins where y has a nonzero value when $x = 0$. The value y has a "head start."

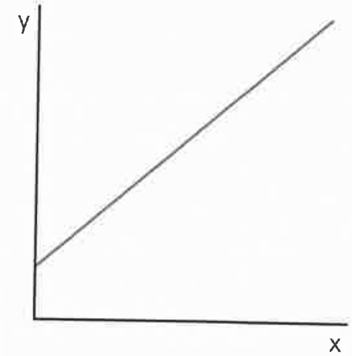


FIGURE C.1

Figure C.2 shows a graph of the equation $v = v_0 + gt$. Speed v is plotted along the y-axis, and time t along the x-axis. As you can see, there is a linear relationship between v and t. Note that the initial speed is 10 m/s. If the initial speed were 0, as in dropping an object from rest, then the graph would intercept the origin, where both v and t are 0. Note that the graph originates at $v = 10$ m/s when $t = 0$, showing a 10-m/s "head start."

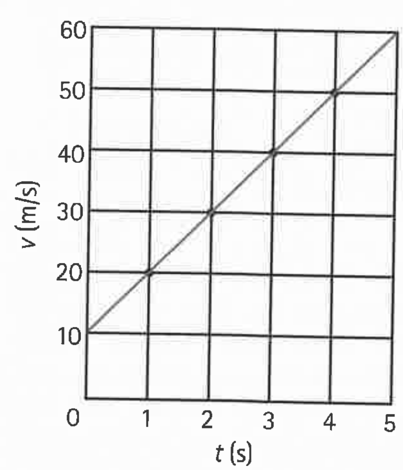


FIGURE C.2

Many physically significant relationships are more complicated than linear relationships, however. If you double the size of a room, the area of the floor increases four times; tripling the size of the room increases the floor area nine times; and so on. This is one example of a *nonlinear*

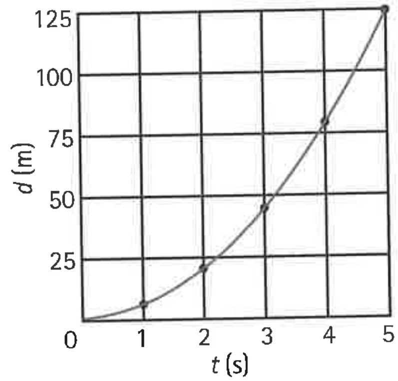


FIGURE C.3

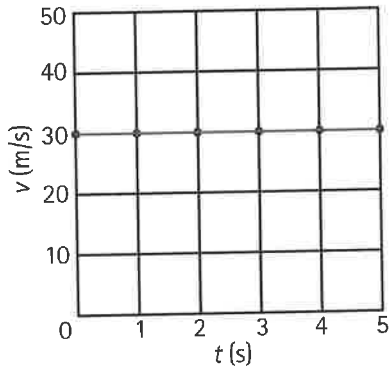


FIGURE C.5

relationship. Figure C.3 shows a graph of another nonlinear relationship: distance versus time in the equation of free fall from rest, $d = \frac{1}{2}gt^2$.

Figure C.4 shows a *radiation curve.* The *curve* (or graph) shows the rather complex nonlinear relationship between intensity I and radiation wavelength λ for a glowing object at 2000 K. The graph shows that radiation is most intense when λ equals about 1.4 µm. Which is brighter, radiation at 0.5 µm or radiation at 4.0 µm? The graph can quickly tell you that radiation at 4.0 µm is appreciably more intense.

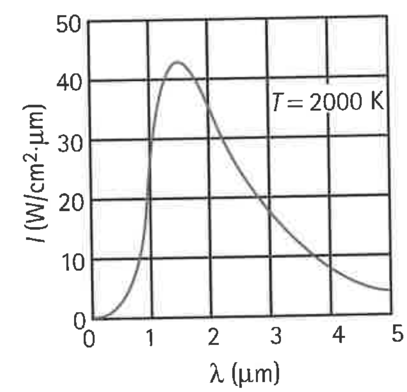


FIGURE C.4

Slope and Area Under the Curve

Quantitative information can be obtained from a graph's *slope* and the *area under the curve.* The slope of the graph in Figure C.2 represents the rate at which v increases relative to t. It can be calculated by dividing a segment Δv along the y-axis by a corresponding segment Δt along the x-axis. For example, dividing Δv of 30 m/s by Δt of 3 s gives $\Delta v/\Delta t$ = 10 m/s·s = 10 m/s^2, the acceleration due to gravity. By contrast, consider the graph in Figure C.5, which is a horizontal straight line. Its slope of zero shows zero acceleration—that is, constant speed. The graph shows that the speed is 30 m/s, acting throughout the entire 5-second interval. The rate of change, or slope, of the speed with respect to time is zero—there is no change in speed at all.

The area under the curve is an important feature of a graph because it often has a physical interpretation. For example, consider the area under the graph of v versus t shown in Figure C.6. The shaded region is a rectangle with sides 30 m/s and 5 s. Its area is 30 m/s × 5 s = 150 m. In this example, the area is the distance covered by an object moving at constant speed of 30 m/s for 5 *s* ($d = vt$).

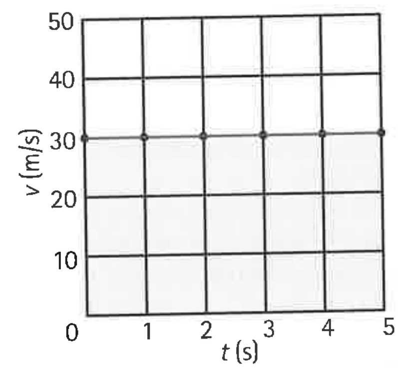


FIGURE C.6

The area need not be rectangular. The area beneath any curve of v versus t represents the distance traveled in a given time interval. Similarly, the area beneath a curve of acceleration versus time gives the change of velocity in a time interval. The area beneath a force-versus-time curve gives the change of momentum. (What does the area beneath a force-versus-distance curve give?) The nonrectangular area under various curves, including rather complicated ones, can be found by way of an important branch of mathematics—*integral calculus.*

Graphing with Conceptual Physics

You may develop basic graphing skills in the laboratory part of this course. The lab "Blind as a Bat" introduces you to graphing concepts. It also gives you a chance to work with a computer and sonic-ranging device. The lab "Trial and Error" will show you the useful technique of converting a nonlinear graph to a linear one to discover a direct proportionality. The area under the curve is the

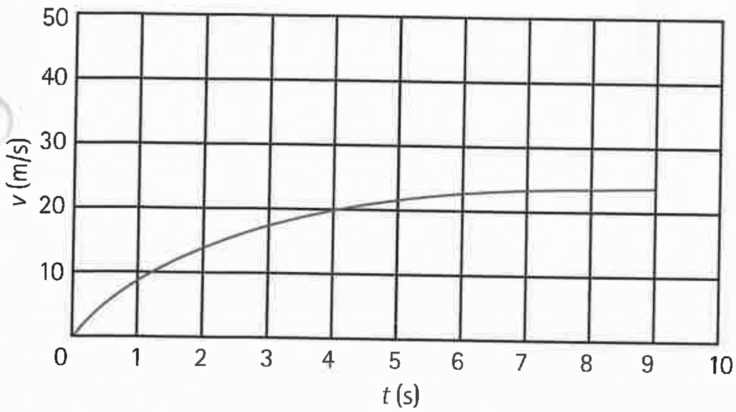


FIGURE C.7

basis of the lab experiment "Impact Speed." You will learn more about graphing in other labs as well.

You may also learn in the lab part of your *Conceptual Physics* course that computers can graph data for you. You are not being lazy when you graph your data with a software program. Instead of investing time and energy scaling the axes and plotting points, you spend your time and energy investigating the meaning of the graph, a high level of thinking!

CHECK POINT

Figure C.7 is a graphical representation of a ball dropped into a mine.

1. How long did the ball take to hit the bottom?
2. What was the ball's speed when it struck bottom?
3. What does the decreasing slope of the graph tell you about the acceleration of the ball with increasing speed?
4. Did the ball reach terminal speed before hitting the bottom of the shaft? If so, about how many seconds did it take to reach its terminal speed?
5. What is the approximate depth of the mine shaft?

Check Your Answers

1. 9 s
2. 25 m/s
3. Acceleration decreases as speed increases (due to air resistance).
4. Yes (since slope curves to zero), about 7 s.
5. Depth is about 170 m. (The area under the curve is about 17 squares, each of which represents 10 m.)

APPENDIX C ONLINE RESOURES

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Tutorial

- Graphing

Appendix D

MORE ABOUT VECTORS

Vectors and Scalars

A *vector* quantity is a directed quantity—one that must be specified not only by magnitude (size) but by direction as well. Recall, from Chapter 5, that velocity is a vector quantity. Other examples are force, acceleration, and momentum. In contrast, a *scalar* quantity can be specified by magnitude alone. Some examples of scalar quantities are speed, time, temperature, and energy.

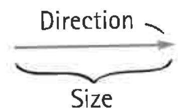


FIGURE D.1

Vector quantities may be represented by arrows. The length of the arrow tells you the magnitude of the vector quantity, and the arrowhead tells you the direction of the vector quantity. Such an arrow drawn to scale and pointing appropriately is called a *vector.*

Adding Vectors

Vectors that add together are called *component vectors.* The sum of component vectors is called a *resultant.*

To add two vectors, make a parallelogram with two component vectors acting as two of the adjacent sides (Figure D.2). (Here our parallelogram is a rectangle.) Then draw a diagonal from the origin of the vector pair; this is the resultant (Figure D.3).

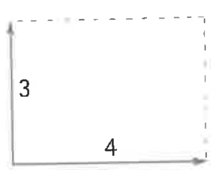


FIGURE D.2

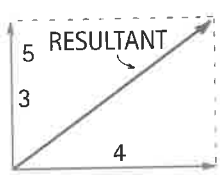


FIGURE D.3

Caution: Do not try to mix vectors! We cannot add apples and oranges, so velocity vectors combine only with velocity vectors, force vectors combine only with force vectors, and acceleration vectors combine only with acceleration vectors—each on its own vector diagram. If you ever show different kinds of vectors on the same diagram, use different colors or some other method of distinguishing the different kinds of vectors.

Finding Components of Vectors

Recall, from Chapter 5, that to find a pair of perpendicular components for a vector, first draw a dashed line through the tail of the vectors (in the direction of one of the desired components). Second, draw another dashed line through the tail end of the vector at right angles to the first dashed line. Third, make a rectangle whose diagonal is the given vector. Draw in the two components. Here we let **F** stand for "total force," **U** stand for "upward force," and **S** stand for "sideways force."

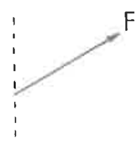


FIGURE D.4

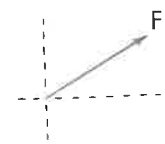


FIGURE D.5

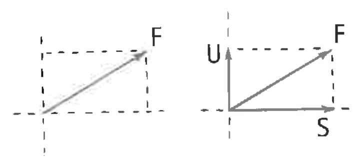


FIGURE D.6

EXAMPLES

1. Ernie Brown pushes a lawnmower and applies a force that pushes it forward and also against the ground. In Figure D.7, F represents the force applied by Ernie. We can separate this force into two components. The vector D represents the downward component, and S is the sideways component, the force that moves the lawnmower forward. If we know the magnitude and direction of the vector F, we can estimate the magnitude of the components from the vector diagram.

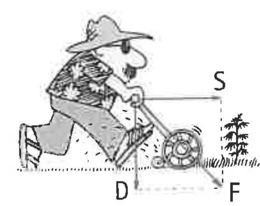


FIGURE D.7

2. Would it be easier to push or pull a wheelbarrow over a step? Figure D.8 shows the force at the wheel's center. When you push a wheelbarrow, part of the force is directed downward, which makes it harder to get over the step. When you pull, however, part of the pulling force is directed upward, which helps to lift the wheel over the step. Note that the vector diagram suggests that pushing the wheelbarrow may not get it over the step at all. Do you see that the height of the

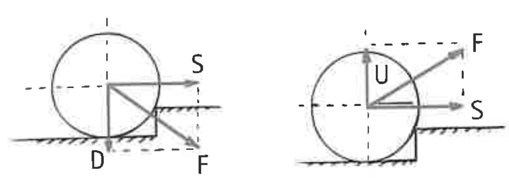


FIGURE D.8

step, the radius of the wheel, and the angle of the applied force determine whether the wheelbarrow can be pushed over the step? We see how vectors help us analyze a situation so that we can see just what the problem is!

3. If we consider the components of the weight of an object rolling down an incline, we can see why its speed depends on the angle. Note that the steeper the incline, the greater the component **S** becomes and the faster the object rolls. When the incline is vertical, **S** becomes equal to the weight, and the object attains maximum acceleration, $9.8\ m/s^2$. There are two more force vectors that are not shown: the normal force **N**, which is equal and oppositely directed to **D**, and the friction force **f**, acting at the barrel-plane contact.

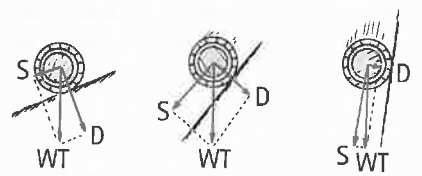


FIGURE D.9

4. When moving air strikes the underside of an airplane wing, the force of air impact against the wing may be represented by a single vector perpendicular to the plane of the wing (Figure D.10). We represent the force vector as acting midway along the lower wing surface, where the dot is, and pointing above the wing to show the direction of the resulting wind impact force. This force can be broken up into two components, one sideways and the other up. The upward component, U, is called *lift*. The sideways component, S, is called *drag*. If the aircraft is to fly at constant velocity at constant altitude, then lift must equal the weight of the aircraft and the thrust of the plane's engines must equal drag. The magnitude of lift (and drag) can be altered by changing the speed of the airplane or by changing the angle (called *angle of attack*) between the wing and the horizontal.

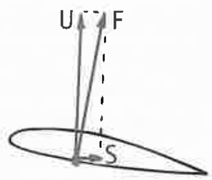


FIGURE D.10

5. Consider the satellite moving clockwise in Figure D.11. Everywhere in its orbital path, gravitational force **F** pulls it toward the center of the host planet. At position A we see **F** separated into two components: **f**, which is tangent to the path of the projectile, and **f′**, which is perpendicular to the path. The relative magnitudes of these components in comparison to the magnitude of **F** can be seen in the imaginary rectangle they compose; **f** and **f′** are the sides, and **F** is the diagonal. We see that component **f** is along the orbital path but against the direction of motion of the satellite. This force component reduces the speed of the satellite. The other component, **f′**, changes the direction of the satellite's motion and pulls it away from its tendency to go in a straight line. So the path of the satellite curves. The satellite loses speed until it reaches position B. At this farthest point from the planet (apogee), the gravitational force is somewhat weaker but perpendicular to the satellite's motion, and component **f** has reduced to zero. Component **f′**, on the other hand, has increased and is now fully merged to become **F**. Speed at this point is not enough for circular orbit, and the satellite begins to fall toward the planet. It picks up speed because the component **f** reappears and is in the direction of motion as shown in position C. The satellite picks up speed until it whips around to position D (perigee), where once again the direction of motion is perpendicular to the gravitational force, **f′** blends to full **F**, and **f** is nonexistent. The speed is in excess of that needed for circular orbit at this distance, and it overshoots to repeat the cycle. Its loss in speed in going from D to B equals its gain in speed from B to D. Kepler discovered that planetary paths are elliptical, but never knew why. Do you?

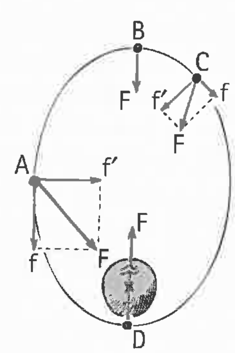


FIGURE D.11

6. Refer to the Polaroids held by Ludmila back in Chapter 29, in Figure 29.35. In the first picture (a), we see that light is transmitted through the pair of Polaroids because their axes are aligned. The emerging light can be represented as a vector aligned with the polarization axes of the Polaroids. When the Polaroids are crossed (b), no light emerges because light passing through the first Polaroid is perpendicular to the polarization axes of the second Polaroid, with no components along its axis. In the third picture (c), we see that light is transmitted when a third Polaroid is sandwiched at an angle between the crossed Polaroids. The explanation for this is shown in Figure D.12.

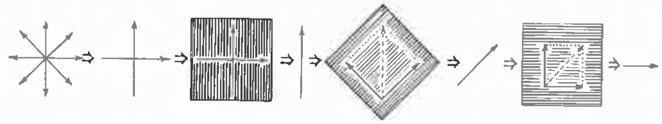


FIGURE D.12

Sailboats

Sailors have always known that a sailboat can sail downwind, in the direction of the wind. Sailors have not always known, however, that a sailboat can sail upwind, against the wind. One reason for this has to do with a feature that is common only to recent sailboats—a fin-like keel that extends deep beneath the bottom of the boat to ensure that the boat will knife through the water only in a forward (or backward) direction. Without a keel, a sailboat could be blown sideways.

Figure D.13 shows a sailboat sailing directly downwind. The force of wind impact against the sail accelerates the boat. Even if the drag of the water and all other resistance forces are negligible, the maximum speed of the boat is the wind speed. This is because the wind will not make impact against the sail if the boat is moving as fast as the wind. The wind would have no speed relative to the boat and the sail would simply sag. With no force, there is no acceleration. The force vector in Figure D.13 *decreases* as the boat travels

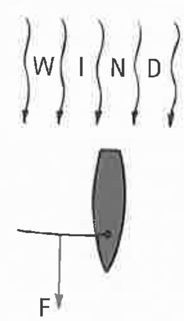


FIGURE D.13

faster. The force vector is maximum when the boat is at rest and the full impact of the wind fills the sail, and is minimum when the boat travels as fast as the wind. If the boat is somehow propelled to a speed faster than the wind (by way of a motor, for example), then air resistance against the front side of the sail will produce an oppositely directed force vector. This will slow the boat down. Hence, the boat when driven only by the wind cannot exceed wind speed.

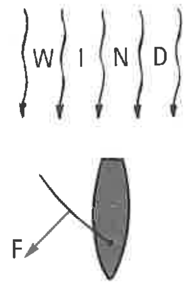


FIGURE D.14

If the sail is oriented at an angle, as shown in Figure D.14, the boat will move forward, but with less acceleration. There are two reasons for this:

1. The force on the sail is less because the sail does not intercept as much wind in this angular position.
2. The direction of the wind impact force on the sail is not in the direction of the boat's motion but is perpendicular to the surface of the sail. Generally speaking, whenever any fluid (liquid or gas) interacts with a smooth surface, the force of interaction is perpendicular to the smooth surface.* The boat does not move in the same direction as the perpendicular force on the sail, but is constrained to move in a forward (or backward) direction by its keel.

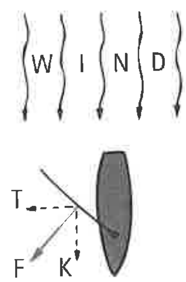


FIGURE D.15

We can better understand the motion of the boat by resolving the force of wind impact, **F**, into perpendicular components. The important component is that which is parallel to the keel, which we label **K**, and the other component is perpendicular to the keel, which we label **T**. It is the component **K**, as shown in Figure D.15, that is responsible for the forward motion of the boat. Component **T** is a useless force that tends to tip the boat over and move it sideways. This component force is offset by the deep keel. Again, maximum speed of the boat can be no greater than wind speed.

Many sailboats sailing in directions other than exactly downwind (Figure D.16) with their sails properly oriented can exceed wind speed. In the case of a sailboat cutting across the wind, the wind may continue to make impact with the sail even after the boat exceeds wind speed. A surfer, in a similar way, exceeds the velocity of the propelling wave by angling his surfboard across the wave. Greater angles to the propelling medium (wind for the boat, water wave for the surfboard) result in greater speeds. A sailcraft can sail faster cutting across the wind than it can sailing downwind.

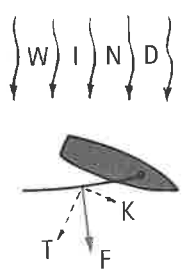


FIGURE D.16

As strange as it may seem, maximum speed for most sailcraft is attained by cutting into (against) the wind, that is, by angling the sailcraft in a direction upwind! Although a sailboat cannot sail directly upwind, it can reach a destination upwind by angling back and forth in a zigzag fashion. This is called *tacking*. Suppose the boat and sail are as shown in Figure D.17. Component K will push the boat along in a forward direction, angling into the wind. In the position shown, the boat can sail faster than the speed of the wind. This is because as the boat travels faster, the impact of wind is increased. This is similar to running in a rain that comes down at an angle. When you run into the direction of the downpour, the drops strike you harder and more frequently, but when you run away from the direction of the downpour, the drops don't strike you as hard or as frequently. In the same way, a boat sailing upwind experiences greater wind impact force, while a boat sailing downwind experiences a decreased wind impact force. In any case, the boat reaches its terminal speed when opposing forces cancel the force of wind impact. The opposing forces consist mainly of water resistance against the hull of the boat. The hulls of racing boats are shaped to minimize this resistive force, which is the principal deterrent to high speeds.

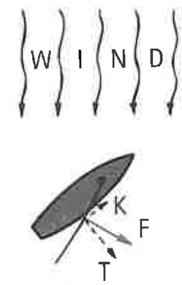
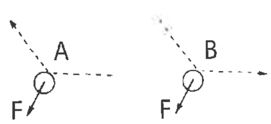


FIGURE D.17

Iceboats (sailcraft equipped with runners for traveling on ice) encounter no water resistance and can travel at several times the speed of the wind when they tack upwind. Although ice friction is nearly absent, an iceboat does not accelerate without limits. The terminal velocity of a sailcraft is determined not only by opposing friction forces but also by the change in relative wind direction. When the boat's orientation and speed are such that the wind seems to shift in direction, so the wind moves parallel to the sail rather than into it, forward acceleration ceases—at least in the case of a flat sail. In practice, sails are curved and produce an airfoil that is as important to sailcraft as it is to aircraft, as discussed in Chapter 14.



*You can do a simple exercise to see that this is so. Try bouncing a coin off another on a smooth surface, as shown. Note that the struck coin moves at right angles (perpendicular) to the contact edge. Note also that it makes no difference whether the projected coin moves along path A or path B. See your instructor for a more rigorous explanation, which involves momentum conservation.

APPENDIX D ONLINE RESOURES

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Tutorial

- Vectors

Appendix E

EXPONENTIAL GROWTH AND DOUBLING TIME*

One of the most important things we seem unable to perceive is the process of exponential growth. We think we understand how compound interest works, but we can't get it through our heads that a fine piece of tissue paper folded upon itself 50 times (if that were possible) would be more than 20 million kilometers thick. If we could, we could "see" why our income buys only half of what it did 4 years ago, why the price of everything has doubled in the same time, why populations and pollution proliferate out of control.**

When a quantity such as money in the bank, population, or the rate of consumption of a resource steadily grows at a fixed percent per year, we say the growth is exponential. Money in the bank may grow at 4 percent per year; electric power generating capacity in the United States grew at about 7 percent per year for the first three-quarters of the 20th century. The important thing about exponential growth is that the time required for the growing quantity to double in size (increase by 100 percent) is also constant. For example, if the population of a growing city takes 12 years to double from 10,000 to 20,000 inhabitants and its growth remains steady, in the next 12 years the population will double to 40,000, and in the next 10 years to 80,000, and so on.

There is an important relationship between the percent growth rate and its *doubling time*, the time it takes to double a quantity:†

$$\text{Doubling time} = \frac{69.3}{\text{percent growth per unit time}} \approx \frac{70}{\%}$$

So to estimate the doubling time for a steadily growing quantity, we simply divide the number 70 by the percentage growth rate. For example, the 7 percent growth rate of electric power generating capacity in the United States means that in the past the capacity had doubled every 10 years [70%/(7%/year) = 10 years]. A 2 percent growth rate for world population means the population of the world doubles every 35 years [70%/(2%/year) = 35 years]. A city planning commission that accepts what seems like a modest 3.5 percent growth rate may not realize that this means that doubling will occur in 70/3.5, or 20 years; that's double capacity for such things as water supply, sewage-treatment plants, and other municipal services every 20 years.

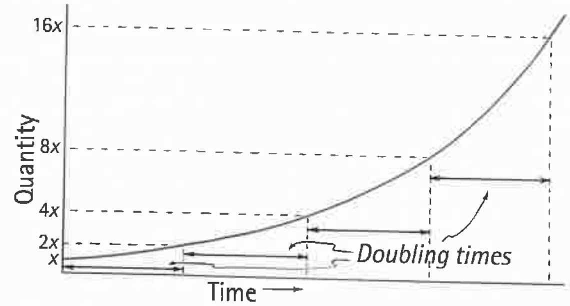


FIGURE E.1

An exponential curve. Notice that each of the successive equal time intervals noted on the horizontal scale corresponds to a doubling of the quantity indicated on the vertical scale. Such an interval is called the doubling time.

What happens when you put steady growth in a finite environment? Consider the growth of bacteria that grow by division, so that one bacterium becomes two, the two divide to become four, the four divide to become eight, and so on. Suppose the division time for a certain strain of bacteria is 1 minute. This is then steady growth—the number of bacteria grows exponentially with a doubling time of 1 minute. Further, suppose that one bacterium is put in a bottle at 11:00 A.M. and that growth continues steadily until the bottle becomes full of bacteria at 12 noon. Consider seriously the following question.

FIGURE E.2

CHECK POINT

When was the bottle half-full?

Check Your Answer

11:59 A.M.; the bacteria will double in number every minute!

*This appendix is adapted from material written by University of Colorado physics professor Albert A. Bartlett, who asserts, "The greatest shortcoming of the human race is our inability to understand the exponential function." See more on Al Bartlett on the web.

**K. C. Cole, *Sympathetic Vibrations* (New York: Morrow, 1984).

†For exponential decay we speak about half-life, the time required for a quantity to reduce to half its value. This case is treated in Chapter 33.

TABLE E.1

The Last Minutes in the Bottle

Time	Part Full (%)	Part Empty
11:54 A.M.	1/64 (1.5%)	63/64
11:55 A.M.	1/32 (3%)	31/32
11:56 A.M.	1/16 (6%)	15/16
11:57 A.M.	1/8 (12%)	7/8
11:58 A.M.	1/4 (25%)	3/4
11:59 A.M.	1/2 (50%)	1/2
12:00 noon	full (100%)	none

TABLE E.2

Effects of the Discovery of Three New Bottles

Time	Effect
11:58 A.M.	Bottle 1 is 1/4 full
11:59 A.M.	Bottle 1 is 1/2 full
12:00 noon	Bottle 1 is full
12:01 P.M.	Bottles 1 and 2 are both full
12:02 P.M.	Bottles 1, 2, 3, and 4 are all full

It is startling to note that at 2 minutes before noon the bottle was only 1/4 full. Table E.1 summarizes the amount of space left in the bottle in the last few minutes before noon. If you were an average bacterium in the bottle, at which time would you first realize that you were running out of space? For example, would you sense there was a serious problem at 11:55 A.M., when the bottle was only 3% filled (1/32) and had 97% of open space (just yearning for development)? The point here is that there isn't much time between the moment that the effects of growth become noticeable and the time when they become overwhelming.

Suppose that at 11:58 A.M. some farsighted bacteria see that they are running out of space and launch a full-scale search for new bottles. Luckily, at 11:59 A.M. they discover three new empty bottles, three times as much space as they had ever known. This quadruples the total resource space ever known to the bacteria, for they now have a total of four bottles, whereas before the discovery they had only one. Further suppose that, thanks to their technological proficiency, they are able to migrate to their new habitats without difficulty. Surely, it seems to most of the bacteria that their problem is solved—and just in time.

CHECK POINT

If the bacteria growth continues at the unchanged rate, what time will it be when the three new bottles are filled to capacity?

Check Your Answer

12:02 P.M.!

We see from Table E.2 that quadrupling the resource extends the life of the resource by only two doubling times. In our example the resource is space—but it could as well be coal, oil, uranium, or any nonrenewable resource.

Continued growth and continued doubling lead to enormous numbers. In two doubling times, a quantity will double twice ($2^2 = 4$; quadruple) in size; in three doubling times, its size will increase eightfold ($2^3 = 8$); in four doubling times, it will increase sixteenfold ($2^4 = 16$); and so on.

This is best illustrated by the story of the court mathematician in India who years ago invented the game of chess for his king. The king was so pleased with the game that he offered to repay the mathematician, whose request seemed modest enough. The mathematician requested a single grain of wheat on the first square of the chessboard, two grains on the second square, four on the third square, and so on, doubling the number of grains on each succeeding square until all squares had been used. At this rate there would be 2^{63} grains of wheat on the 64th square. The king soon saw that he could not fill this "modest" request, which amounted to more wheat than had been harvested in the entire history of Planet Earth!

FIGURE E.3

A single grain of wheat placed on the first square of the chessboard is doubled on the second square, this number is doubled on the third, and so on, presumably for all 64 squares. Note that each square contains one more grain than all the preceding squares combined. Does enough wheat exist in the world to fill all 64 squares in this manner?

It is interesting and important to note that the number of grains on any square is one grain more than the total of all grains on the preceding squares. This is true anywhere on the board. Note from Table E.3 that when eight grains are placed on the fourth square, the eight is one more than the total of seven grains that were already on the board. Or the 32 grains placed on the sixth square is one more than the total of 31 grains that were already on the board. We see that in one doubling time we use more than all that had been used in all the preceding growth!

So if we speak of doubling energy consumption in the next however many years, bear in mind that this means in these years we will consume more energy than has

TABLE E.3
Filling the Squares on the Chessboard

Square Number	Grains on Square	Total Grains Thus Far
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
.	.	.
.	.	.
.	.	.
64	2^{63}	$2^{64} - 1$

heretofore been consumed during the entire preceding period of steady growth. And if power generation continues to use predominantly fossil fuels, then except for some improvements in efficiency, we would burn up in the next doubling time a greater amount of coal, oil, and natural gas than has already been consumed by previous power generation, and except for improvements in pollution control, we can expect to discharge even more toxic wastes into the environment than the millions upon millions of tons already discharged over all the previous years of industrial civilization. We would also expect more human-made calories of heat to be absorbed by Earth's ecosystem than have been absorbed in the entire past! At the previous 7% annual growth rate in energy production, all this would occur in one doubling time of a single decade. If over the coming years the annual growth rate remains at half this value, 3.5 percent, then all this would take place in a doubling time of two decades. Clearly this cannot continue!

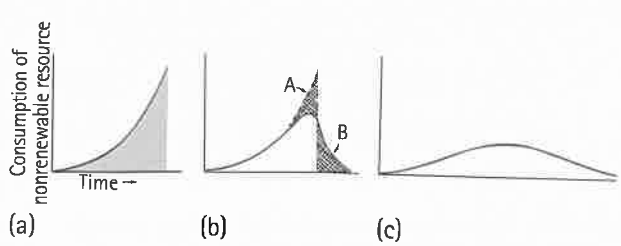
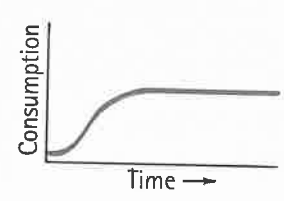


FIGURE E.4
(a) If the exponential rate of consumption for a nonrenewable resource continues until it is depleted, consumption falls abruptly to zero. The shaded area under this curve represents the total supply of the resource. (b) In practice, the rate of consumption levels off and then falls less abruptly to zero. Note that the crosshatched area A is equal to the crosshatched area B. Why? (c) At lower consumption rates, the same resource lasts a longer time.

The consumption of a nonrenewable resource cannot grow exponentially for an indefinite period, because the resource is finite and its supply finally expires. The most drastic way this could happen is shown in Figure E.4(a), where the rate of consumption, such as barrels of oil per year, is plotted against time, say in years. In such a graph the area under the curve represents the supply of the resource. We see that when the supply is exhausted, the consumption ceases altogether. This sudden change is rarely the case, for the rate of extracting the supply falls as it becomes more scarce. This is shown in Figure E.4(b). Note that the area under the curve is equal to the area under the curve in (a). Why? Because the total supply is the same in both cases. The principal difference is the time taken to finally extinguish the supply. History shows that the rate of production of a nonrenewable resource rises and falls in a nearly symmetric manner, as shown in (c). The time during which production rates rise is approximately equal to the time during which these rates fall to zero or near zero.

Production rates for all nonrenewable resources decrease sooner or later. Only production rates for renewable resources, such as agriculture or forest products, can be maintained at steady levels for long periods of time (Figure E.5), provided such production does not depend on waning nonrenewable resources such as petroleum. Much of today's agriculture is so petroleum-dependent that it can be said that modern agriculture is simply the process whereby land is used to convert petroleum into food. The implications of petroleum scarcity go far beyond rationing of gasoline for cars or fuel oil for home heating.

FIGURE E.5
A curve showing the rate of consumption of a renewable resource such as agricultural or forest products, where a steady rate of production and consumption can be maintained for a long period, provided this production is not dependent upon the use of a nonrenewable resource that is waning in supply.



The consequences of unchecked exponential growth are staggering. It is important to ask: Is growth really good? In answering this question, bear in mind that human growth is an early phase of life that continues normally through adolescence. Physical growth stops when physical maturity is reached. What do we say of growth that continues in the period of physical maturity? We say that such growth is obesity—or worse, cancer.

QUESTIONS TO PONDER

1. According to a French riddle, a lily pond starts with a single leaf. Each day the number of leaves doubles, until the pond is completely covered by leaves on the 30th day. On what day was the pond half covered? One-quarter covered?
2. In an economy that has a steady inflation rate of 7% per year, in how many years does a dollar lose half its value?
3. At a steady inflation rate of 7%, what will be the price every 10 years for the next 50 years for a theater ticket that now costs $20? For a coat that now costs $200? For a car that now costs $20,000? For a home that now costs $200,000?
4. If the sewage treatment plant of a city is just adequate for the city's current population, how many sewage treatment plants will be necessary 42 years later if the city grows steadily at 5% annually?
5. If world population doubles in 40 years and world food production also doubles in 40 years, how many people then will be starving each year compared to now?
6. Suppose you get a prospective employer to agree to hire your services for wages of a single penny for the first day, 2 pennies for the second day, and double each day thereafter providing the employer keeps to the agreement for a month. What will be your total wages for the month?
7. In the preceding exercise, how will your wages for only the 30th day compare to your total wages for the previous 29 days?
8. If fusion power were harnessed today, the abundant energy resulting would probably sustain and even further encourage our present appetite for continued growth and in a relatively few doubling times produce an appreciable fraction of the solar power input to Earth. Make an argument that the current delay in harnessing fusion is a blessing for the human race.