+ CHAPTER 3 +
PROJECTILES

SO FAR, WE HAVE
BEEN WEIGHING
AND DROPPING
THINGS.

NOW LET'S
SHOOT SOME!

IN ORDER TO UNTANGLE
HORIZONTAL AND
VERTICAL MOTIONS?

THE SIMPLEST PROJECTILE MOTION IS TO PROJECT SOMETHING
SIDEWAYS: DRIVING A CAR OFF A CLIFF OR SHOOTING A
BULLET HORIZONTALLY. THE KEY TO UNDERSTANDING
THIS MOTION IS TO REALIZE THAT GRAVITY ACTS ONLY
VERTICALLY. IT AFFECTS ONLY THE **DOWNWARD**
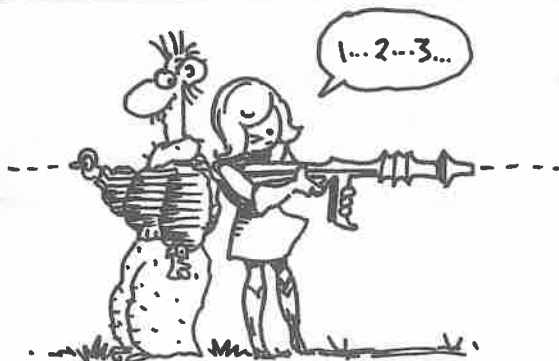PART OF THE MOTION.

NO.
TO DEFEND
OURSELVES
FROM
ALIENS
DISGUISED
AS TARGETS.

FORCE AND
ACCELERATION
ARE DOWNWARD

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THIS FACT IMMEDIATELY ANSWERS A FAMOUS QUESTION: IF RINGO DROPS A BULLET AT THE SAME MOMENT AS I SHOOT A BULLET HORIZONTALLY, WHICH BULLET HITS THE GROUND FIRST? (WE START AT THE SAME HEIGHT.)
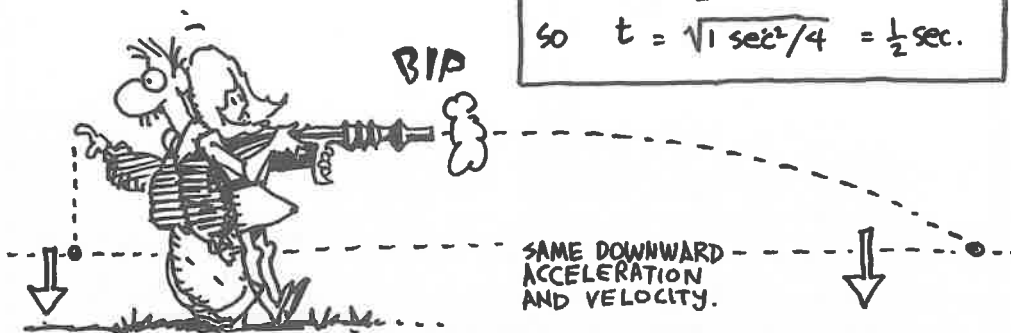
1...2...3...

THEY REACH THE GROUND AT THE SAME TIME, BECAUSE THEY **FALL AT THE SAME RATE.** THE HORIZONTAL MOTION HAS NO EFFECT ON THE VERTICAL MOTION!

EXAMPLE: SUPPOSE I FIRE THE BULLET FROM A SHOULDER HEIGHT OF 4 FT. THEN THE DISTANCE FALLEN IS

$$d = \tfrac{1}{2}gt^2, \quad \text{SO}$$

$$4ft = \tfrac{1}{2}(32ft/sec^2) \cdot t^2$$

$$\text{SO} \quad t = \sqrt{1\ sec^2/4} = \tfrac{1}{2}sec.$$

BIP

SAME DOWNWARD ACCELERATION AND VELOCITY.

IF THE BULLET'S HORIZONTAL SPEED IS 1000 ft/sec, THEN IT GOES 500 FEET IN $\tfrac{1}{2}$ sec

|← 500 ft →|

NOW HERE'S ANOTHER QUESTION: WHAT HAPPENS IF THE GUN IS FIRED UPWARD AT AN ANGLE?

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IN THE ABSENCE OF GRAVITY, THE BULLET WOULD FOLLOW A STRAIGHT LINE FOREVER (NEWTON'S FIRST LAW). WITH GRAVITY, IT **FALLS AWAY** FROM THAT STRAIGHT LINE!
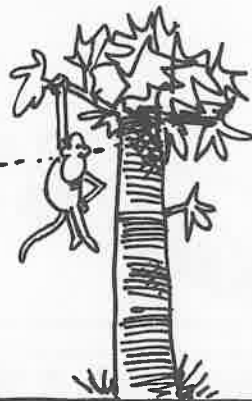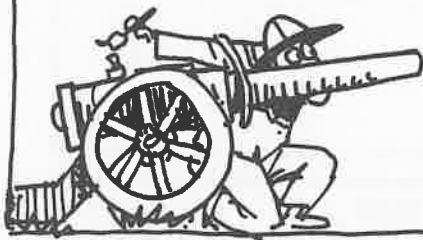
ACCELERATION
g

...WHICH BRINGS US TO A THOUGHT-EXPERIMENT: CALLED

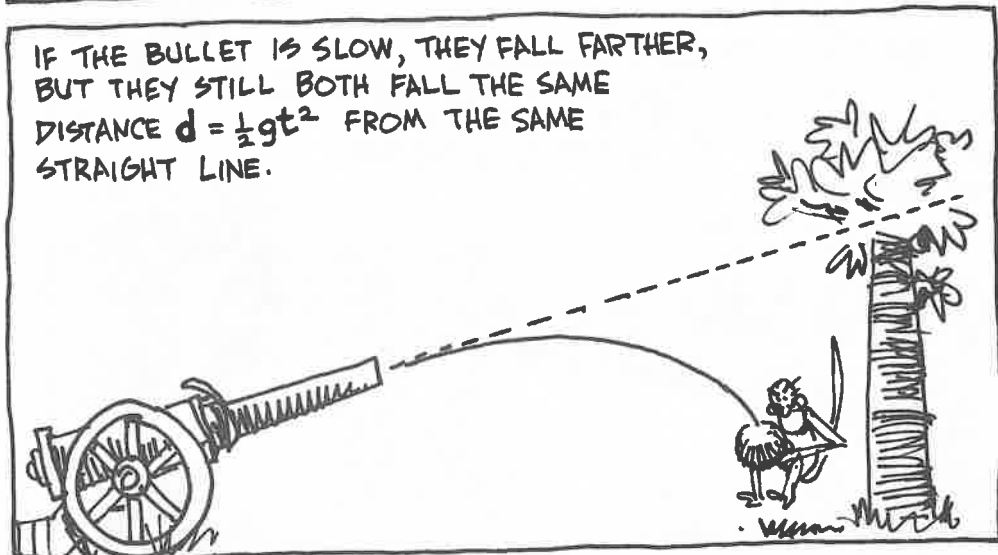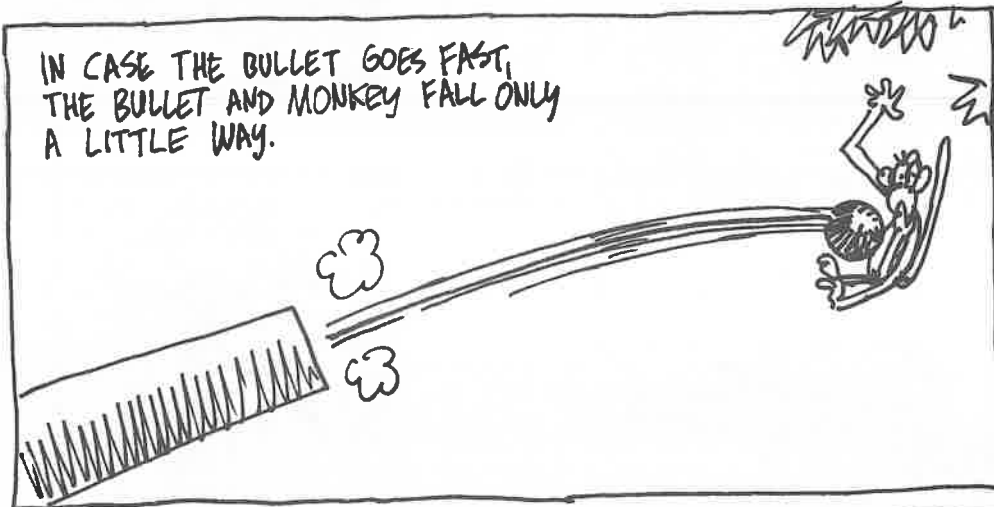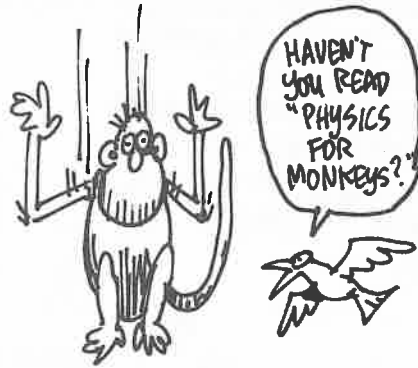# "MONKEY AND HUNTER."

DOWN WITH ANIMAL EXPERIMENTS!

A HUNTER AIMS HIS GUN DIRECTLY AT A MONKEY HANGING FROM A TREE.

THE MONKEY CLEVERLY RELEASES HIS GRIP AT THE EXACT MOMENT THE HUNTER FIRES THE GUN. WHAT HAPPENS?

BWOM

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POOR MONKEY!! IT DOESN'T UNDERSTAND THE INDEPENDENCE OF FALLING AND FORWARD MOTION! BUT YOU DO — SO YOU CAN SEE THAT THE BULLET WILL ALWAYS HIT THE MONKEY!

HAVEN'T YOU READ "PHYSICS FOR MONKEYS?"

IN CASE THE BULLET GOES FAST, THE BULLET AND MONKEY FALL ONLY A LITTLE WAY.

IF THE BULLET IS SLOW, THEY FALL FARTHER, BUT THEY STILL BOTH FALL THE SAME DISTANCE $d = \frac{1}{2}gt^2$ FROM THE SAME STRAIGHT LINE.

· CHAPTER 4 ·
SATELLITE MOTION AND WEIGHTLESS-NESS

NOW WE'RE ON THE MOON, WHERE THERE'S NO AIR RESISTANCE. WATCH AS I FIRE BULLETS HORIZONTALLY WITH GREATER AND GREATER SPEED. EACH BULLET FALLS TO THE GROUND IN THE SAME TIME — THE HORIZONTAL MOTION DOESN'T AFFECT THE FALLING RATE — BUT THE FASTER BULLETS GO FARTHER BEFORE PLOWING INTO THE MOON.
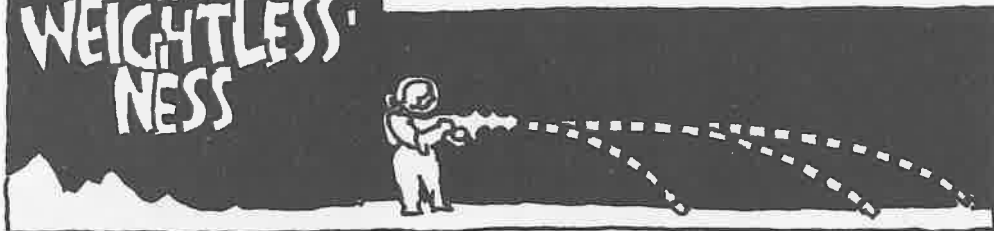
THE GUN IS 4 FEET OFF THE GROUND. ON EARTH, THE BULLET FALLS IN $\frac{1}{2}$ SEC., BUT HERE, WHERE GRAVITY IS WEAKER, IT TAKES 1.2 SEC. (AS LONG AS THE GROUND IS LEVEL).

BUT AS THE BULLETS GO FARTHER, SOMETHING NEW HAPPENS: THE MOON ISN'T FLAT, IT'S ROUND!! THE GROUND STARTS CURVING DOWN UNDER THE BULLET AND AWAY FROM IT.
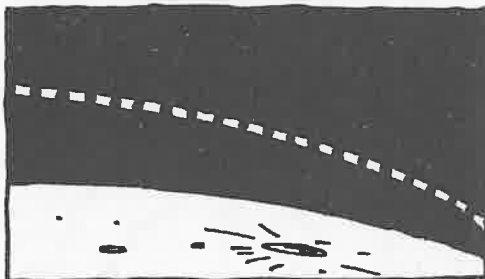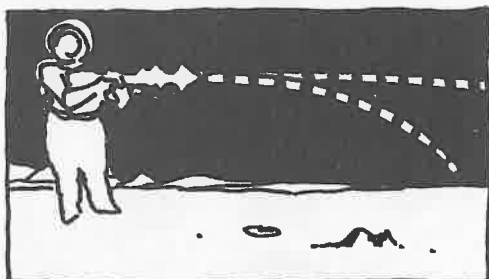
EVENTUALLY, AS I FIRE FASTER AND FASTER, BY THE TIME THE BULLET HAS FALLEN 4 FT., THE GROUND HAS CURVED 4 FT. DOWN AND THE BULLET IS **STILL** 4 FT. HIGH! BY THE TIME IT FALLS ANOTHER 4 FT., SO HAS THE GROUND!

← 4 FT.

THE BULLET IS NOW IN A 4-FOOT-HIGH **ORBIT** AROUND THE MOON. IT IS FALLING CONTINUALLY, BUT THE GROUND IS STEADILY CURVING AWAY BENEATH IT.
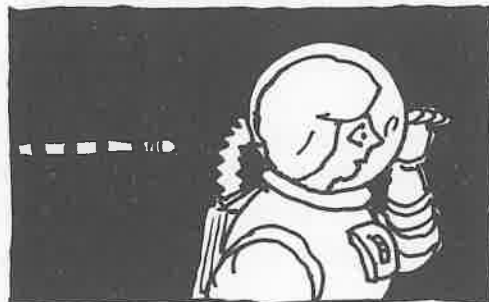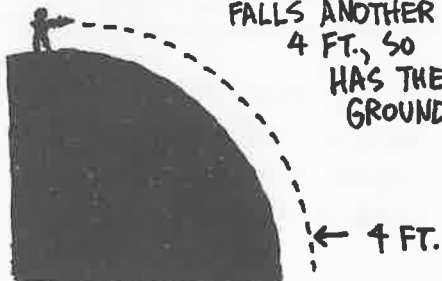
OF COURSE, THIS WORKS ONLY WHEN THERE IS NO AIR
RESISTANCE (AND NO 4·FT·HIGH OBSTACLES!) TO SLOW THE
BULLET, BUT THE EXPERIMENT ILLUSTRATES THE PRINCIPLE
OF SATELLITE MOTION. FROM EARTH WE LAUNCH SATELLITES
ABOVE THE ATMOSPHERE WITH ROCKETS, THEN TILT THEM
OVER AND GIVE THEM ENOUGH **HORIZONTAL** SPEED SO THAT
THE EARTH CURVES AWAY FROM THEM AS THEY FALL.
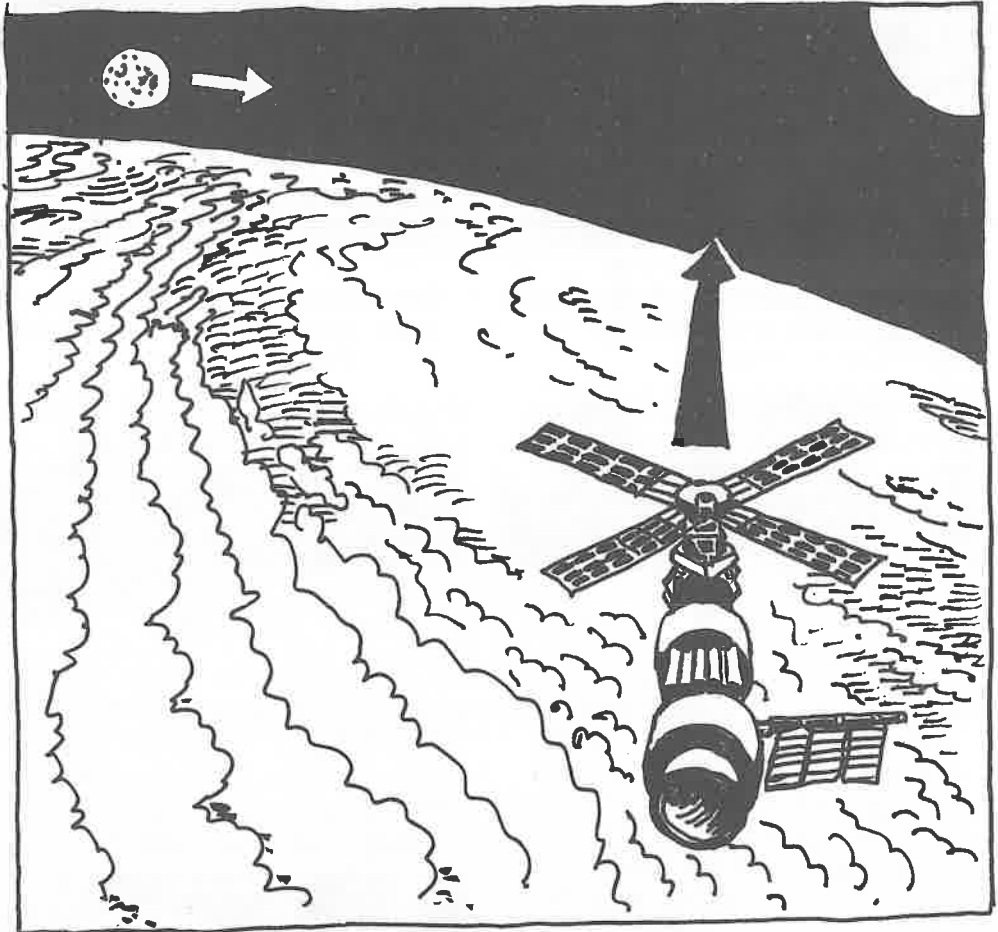
SIMILARLY, OUR NATURAL SATELLITE, THE MOON, FALLS
CONTINUALLY, BUT ITS FORWARD MOTION CARRIES IT ALONG
SO IT REMAINS THE SAME HEIGHT ABOVE EARTH. (THE
MOON'S ORBIT IS CIRCULAR, OR NEARLY SO.)

NOW LET'S GO UP IN THE SPACE SHUTTLE. AS WE REACH ORBITAL SPEED AND I CUT OFF THE ENGINES, THE ONLY FORCE ON US IS **GRAVITY**, AND WE FALL TOWARD EARTH.

WOOAH!
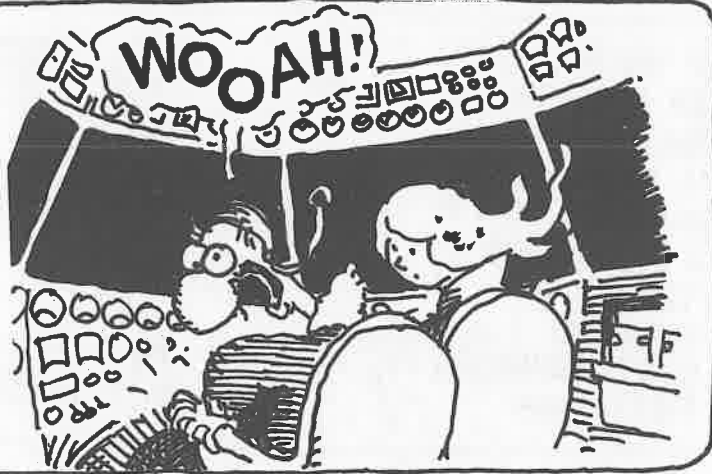
BUT THE SAME IS TRUE OF THE SHUTTLE ITSELF. IT'S ALSO FALLING, AND WITH THE SAME ACCELERATION

YORK

YORK

YORK

SHUTTLE'S ACCELERATION   OUR ACCELERATION
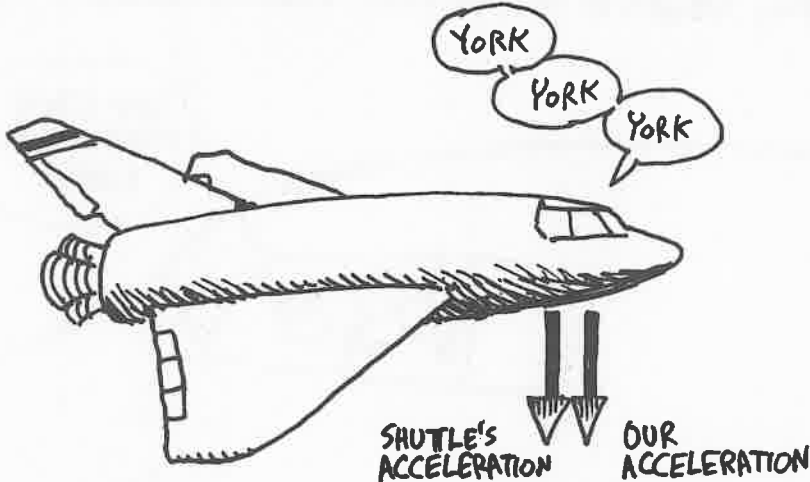
SO THERE IS NO NO RELATIVE MOTION BETWEEN US AND THE SHIP, AND WE FLOAT FREELY INSIDE, WEIGHTLESS !!

STAY IN THE BAG!

IF YOU RELEASE AN APPLE IN THE FALLING SHUTTLE, IT HANGS IN MID-AIR. GIVE IT A NUDGE AND IT TRAVELS IN A STRAIGHT LINE. IT OBEYS NEWTON'S FIRST LAW!

WHENEVER THE ONLY FORCE ON THE CRAFT IS GRAVITY, WHETHER IT'S COASTING UP, FALLING DOWN, OR IN ORBIT, OBJECTS INSIDE ARE **WEIGHTLESS**.

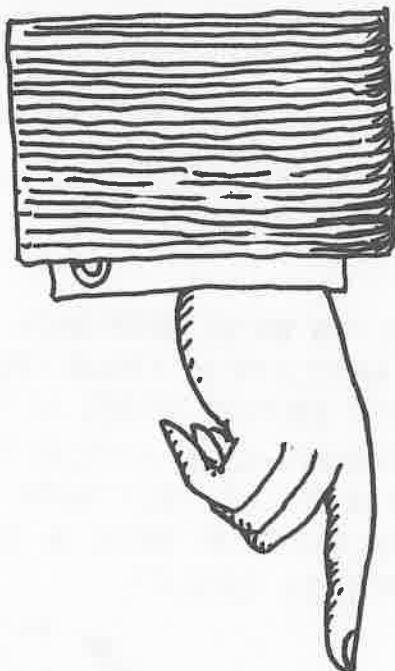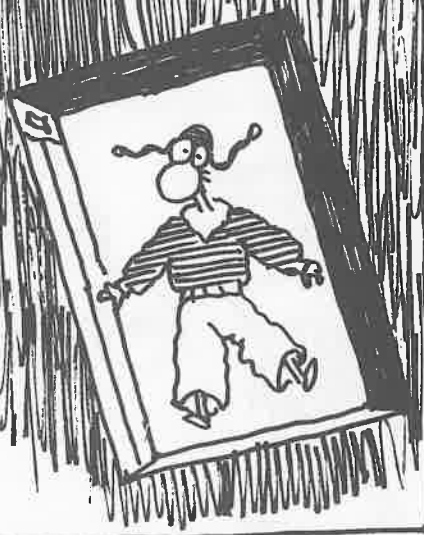THE SCALE KEEPS FLOATING AWAY...
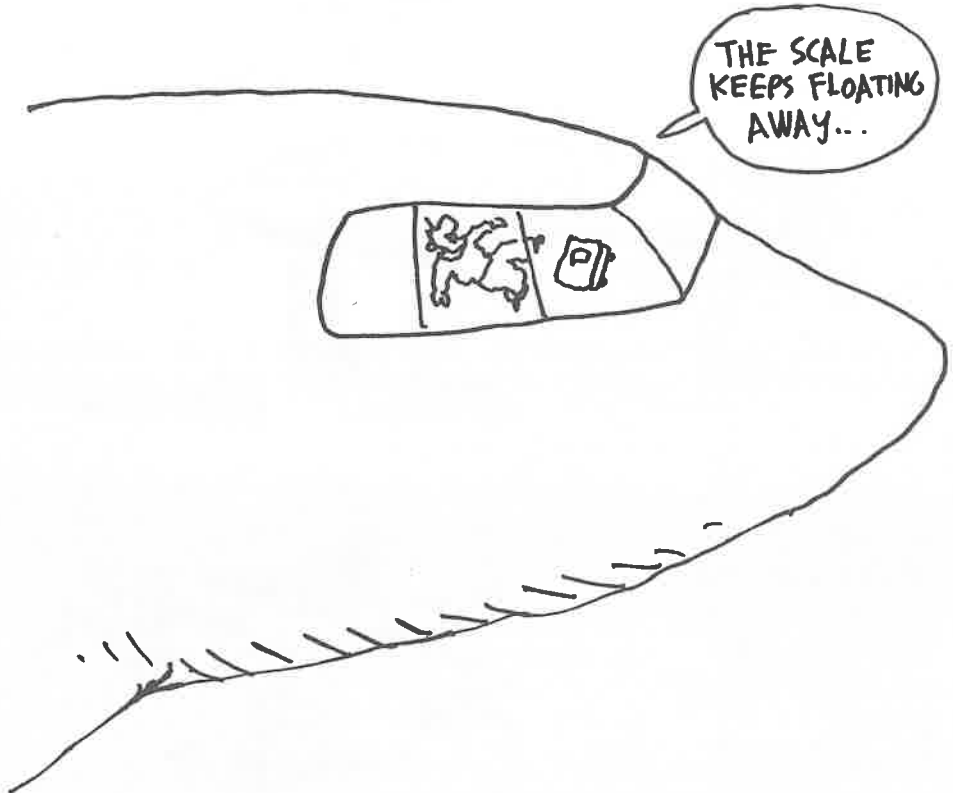
WE CAN DUPLICATE THE
EFFECT HERE ON EARTH.
JUST STEP INTO THIS
ELEVATOR, AND I'LL
CUT THE CABLE !!

YOU'LL
ONLY BE
WEIGHTLESS
A LITTLE
WHILE!

OH,
GOOD.

THUS, ALTHOUGH GRAVITY
PRODUCES ACCELERATION,
NO **ACCELERATION**
FORCES ARE FELT
WITHIN THE FALLING SYSTEM.

THIS WAS ANOTHER HINT TO
EINSTEIN THAT GRAVITY IS A
PROPERTY OF SPACE, RATHER
THAN OBJECTS.

·CHAPTER 5·
OTHER ORBITS

So far we've seen only circular orbits: a satellite is projected horizontally with just enough speed so that it falls away from straight-line motion to match the curvature of a circle. But what would happen if we projected it with a different speed, or at another angle?

BIG V
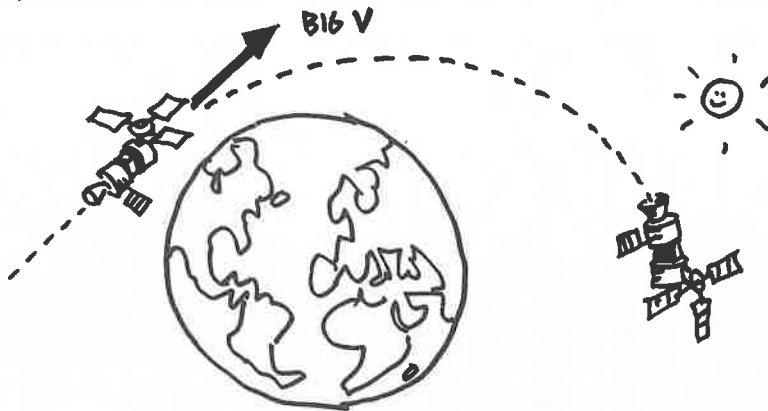
ONE WAY TO WORK OUT THE ORBIT IS WITH A TIME-HONORED MATHEMATICAL TECHNIQUE KNOWN AS "BRUTE FORCE."

ALSO KNOWN AS "UGLY MATH."

M   m
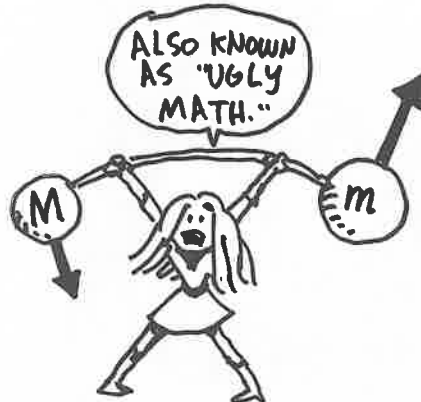
THE BRUTE-FORCE METHOD
STARTS WITH THE GRAVITATIONAL
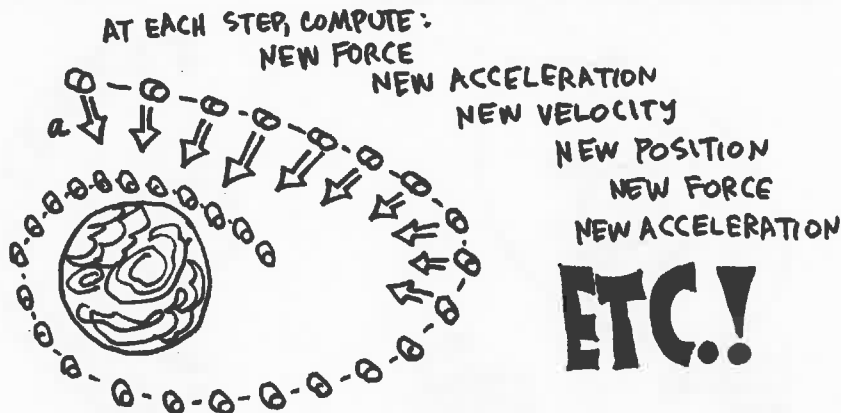FORMULA

$$F = G\frac{Mm}{r^2}.$$

(M = MASS OF EARTH; m = MASS
OF SATELLITE, r = DISTANCE
BETWEEN THEM, G = CONSTANT.)
THIS FORMULA GIVES THE FORCE
ON THE SATELLITE, SO WE CAN
COMPUTE ITS ACCELERATION BY
NEWTON'S SECOND LAW
$a = F/m$. THEN WE CAN
COMPUTE HOW MUCH ITS VELOCITY
CHANGES, OWING TO THIS ACCELERATION.

V_{before}

V_{after}

a a
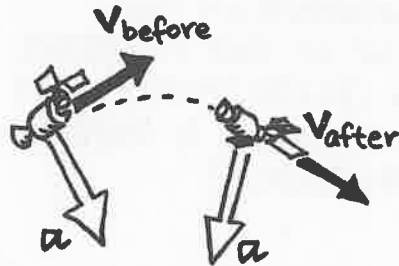
BUT ALAS — AFTER IT HAS MOVED A LITTLE, r IS
DIFFERENT, SO THE GRAVITATIONAL FORCE ON THE
BODY HAS CHANGED! SO WE NEED TO **RE**·CALCULATE
THE ACCELERATION AND NEW VELOCITY FOR THE NEXT
FEW MOMENTS... AND THEN RECALCULATE AGAIN...
AND AGAIN... AND AGAIN...AND AGAIN... THOUSANDS OF
TIMES !!!

AT EACH STEP, COMPUTE:
NEW FORCE
NEW ACCELERATION
NEW VELOCITY
NEW POSITION
NEW FORCE
NEW ACCELERATION

a

ETC.!!

(INCIDENTALLY, THIS STEP-BY-STEP PROCEDURE IS CALLED "NUMERICALLY INTEGRATING A DIFFERENTIAL EQUATION.")
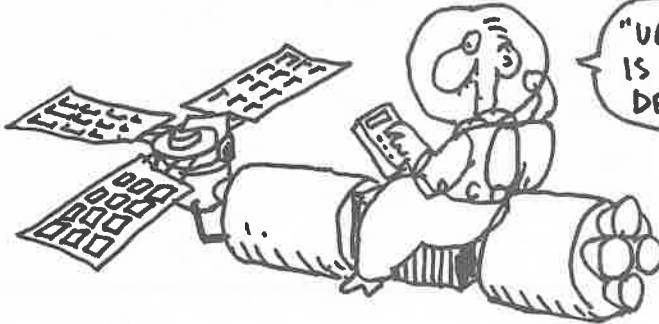
"UGLY MATH" IS MUCH MORE DESCRIPTIVE...

IF THERE ARE ONLY TWO BODIES PRESENT, CALCULUS ALLOWS US TO DERIVE FORMULAS FOR THESE ORBITS. WE FIND THAT THE ONLY POSSIBLE ORBITS IN NEWTON'S GRAVITY ARE CIRCLES, ELLIPSES, PARABOLAS, AND HYPERBOLAS.

SUN

ELLIPTICAL PLANETARY ORBIT

PARABOLIC COMETARY ORBIT

BUT WHEN THERE ARE MORE THAN TWO BODIES, BRUTE FORCE — AND THE COMPUTER — ARE OUR ONLY HOPE! FOR EXAMPLE, THE MOON FOLLOWS A CORKSCREW PATH AROUND THE SUN!

MOON

EARTH

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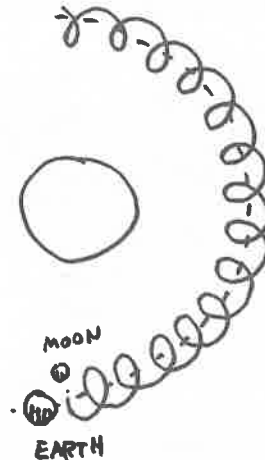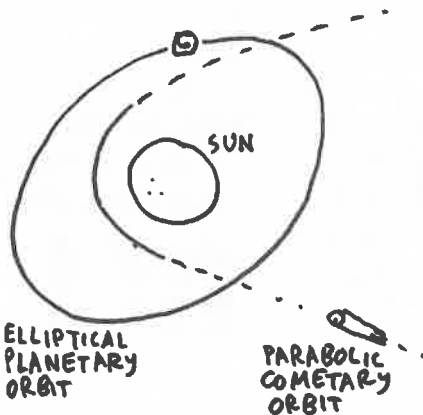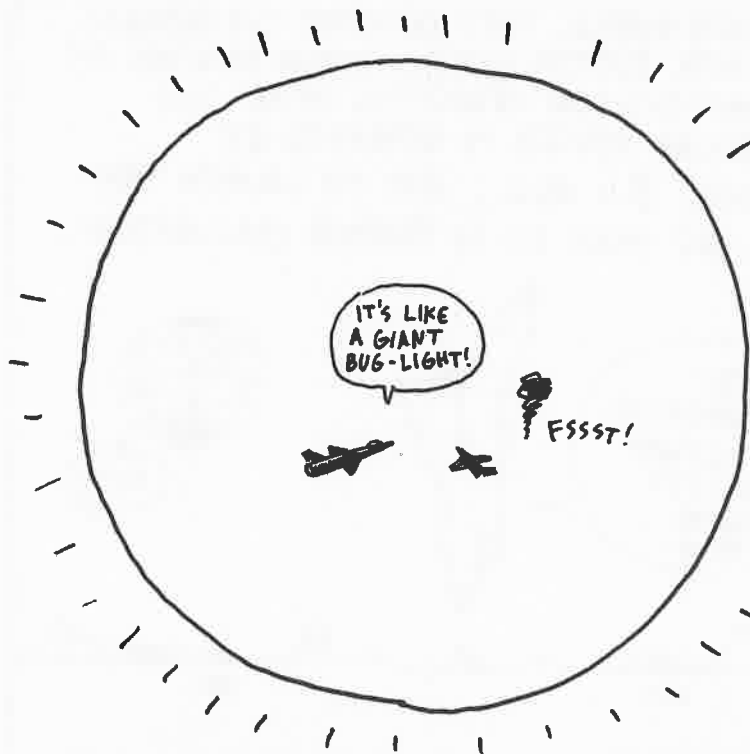
FORTUNATELY, THE SUN IS SO MASSIVE THAT ITS GRAVITY DOMINATES THE SOLAR SYSTEM. AND PLANETARY ORBITS ARE NEARLY EXACT ELLIPSES.

IT'S LIKE A GIANT BUG-LIGHT!

FSSST!

THE FIRST TO SHOW ELLIPTICAL ORBITS WAS KEPLER (1571-1630), WHO PROVED THAT THE ORBIT OF MARS WAS AN ELLIPSE. LATER, NEWTON SHOWED HOW ELLIPTICAL ORBITS RESULT FROM AN INVERSE-SQUARE LAW OF FORCE.
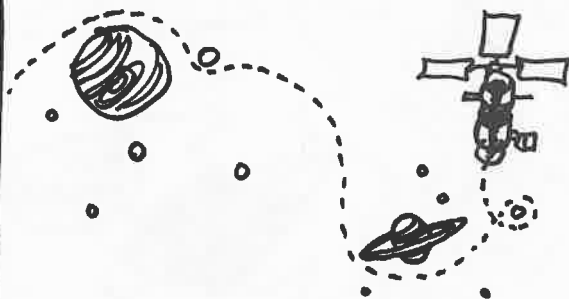
BUT ELLIPSES ARE ONLY APPROXIMATE, AND BRUTE-FORCE MATH IS WHAT WE USE TO SEND OUR SPACE PROBES TO THE PLANETS.

THE EQUATIONS ARE SIMPLE. THEY DESCRIBE THE GENERAL **KIND** OF MOTION, BUT THE ACTUAL MOTION DEPENDS ON THE INITIAL POSITIONS AND VELOCITIES OF ALL THE BODIES. THE SOLAR SYSTEM IS GOVERNED BY $F = G\frac{Mm}{r^2}$ AND $F = ma$, BUT TO LAUNCH THAT SPACE PROBE, WE MUST DO AN IMMENSE CALCULATION.

WE'RE EXPERIENCING A SLIGHT COMPUTER ERROR HERE...

SORRY
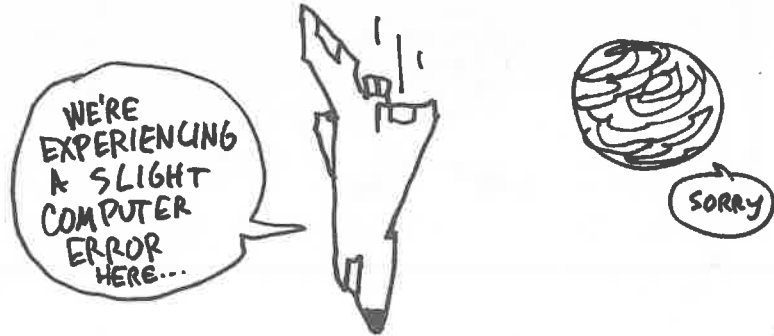
MUCH OF PHYSICS IS LIKE THIS: FIND THE GENERAL EQUATIONS AND SOLVE THEM FOR THE SPECIFIC CASE AT HAND. IS IT POSSIBLE, WE WONDER, TO DESCRIBE ALL THE PHYSICS OF THE UNIVERSE WITH A SMALL LIST OF EQUATIONS STARTING FROM THE INITIAL CONDITIONS OF THE **BIG BANG** ?

WHO KNOWS?