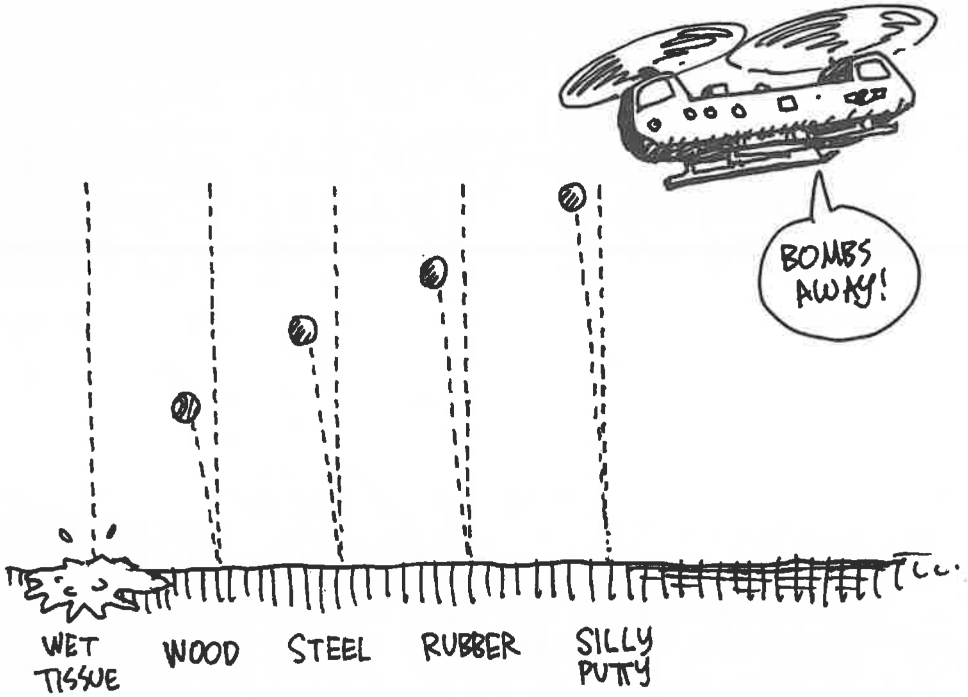
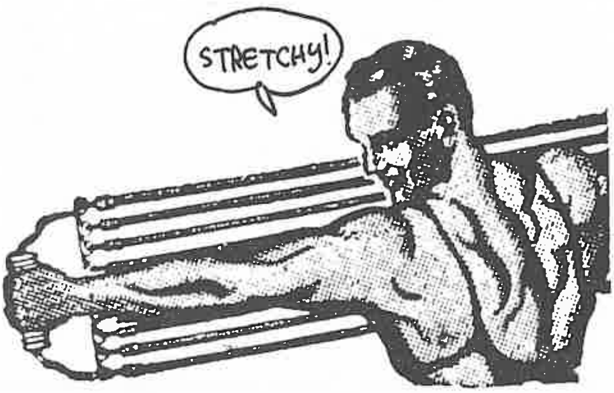
·CHAPTER 10·
COLLISIONS

COLLISIONS PROVIDE GOOD ILLUSTRATIONS OF THE CONSERVATION OF MOMENTUM AND ENERGY. LET'S START BY LETTING SOME THINGS COLLIDE WITH THE GROUND. I'LL DROP SOME BALLS MADE OF VARIOUS MATERIALS, AND SEE HOW HIGH THEY BOUNCE. AS YOU CAN SEE, SOME BOUNCE HIGHER THAN OTHERS.

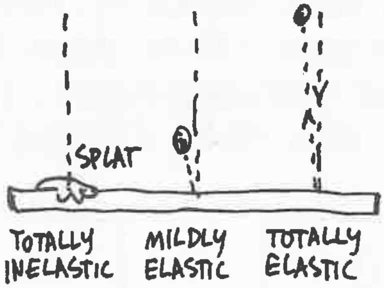


IF A BALL BOUNCES BACK TO THE ORIGINAL HEIGHT, WE SAY IT IS

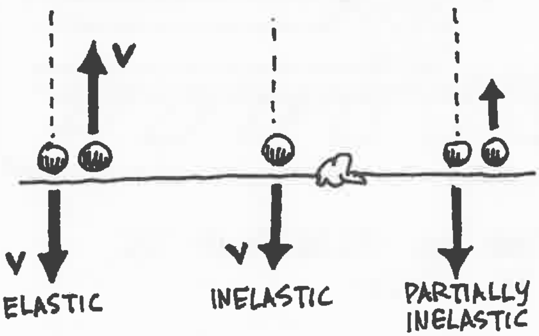
TOTALLY ELASTIC.



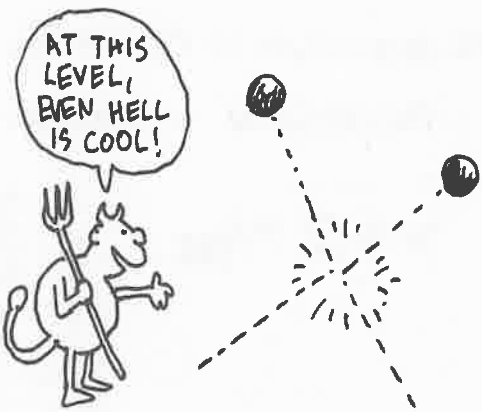
COLLISIONS RANGE FROM TOTALLY ELASTIC TO TOTALLY INELASTIC. IN A TOTALLY INELASTIC COLLISION, THE BALL DOESN'T BOUNCE BACK AT ALL, LIKE THE WAD OF WET TISSUE OR A BLOB OF PUTTY.



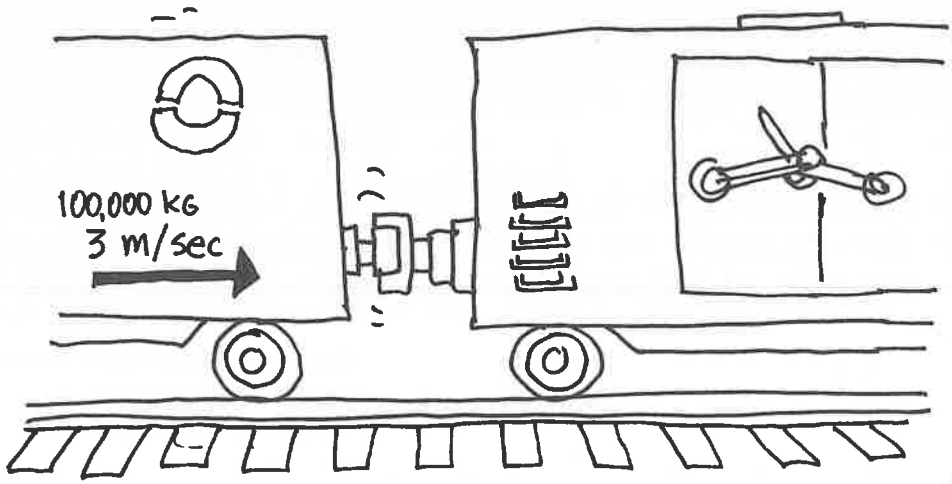
IN A TOTALLY ELASTIC COLLISION, NO KINETIC ENERGY IS LOST AS HEAT ON IMPACT. THE UPWARD SPEED AFTER THE BOUNCE IS THE SAME AS THE DOWNWARD SPEED JUST BEFORE. IN AN INELASTIC COLLISION, SOME OR ALL OF THE OBJECT'S KINETIC ENERGY IS LOST.



YOU MIGHT THINK THAT NO REAL OBJECTS ARE ABSOLUTELY ELASTIC, BUT COLLISIONS BETWEEN ATOMS CAN BE. SINCE HEAT IS THE RANDOM KINETIC ENERGY OF MANY ATOMS, HEAT DOES NOT EXIST AT THE LEVEL OF ONE OR TWO ATOMS!



HERE IS A TOTALLY INELASTIC COLLISION. A LOADED FREIGHT CAR, MASS 100,000 kg, ROLLS AT 3 m/sec INTO A STATIONARY CAR OF MASS 50,000 kg. WHEN THEY HIT, THE COUPLER ENGAGES. (THIS MAKES IT INELASTIC.) THEN WHAT HAPPENS?



SOLUTION: WE WANT TO FIND THE VELOCITY OF THE COUPLED CARS. CALL IT **V**. THEN:

INITIAL MOMENTUM = 100,000 KG × 3 M/SEC

FINAL MOMENTUM = 150,000 KG × **V**

SINCE MOMENTUM IS CONSERVED, THESE TWO ARE EQUAL:

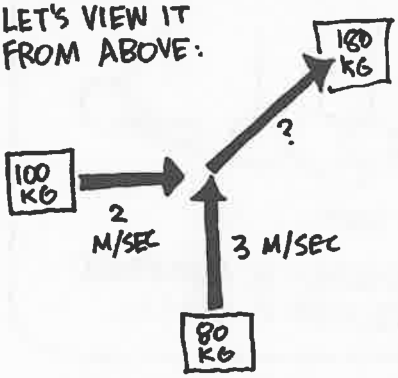
$$150{,}000\ \text{KG} \cdot \mathbf{V} = 300{,}000\ \text{M} \cdot \text{KG/SEC}.$$

SO:

$$\mathbf{V} = 2\ \text{M/SEC}.$$

HERE'S A 2-DIMENSIONAL EXAMPLE: AN 80-KG FOOTBALL PLAYER GOING NORTH AT 3 M/S IS TACKLED BY A 100-KG PLAYER GOING EAST AT 2 M/SEC. AFTER THE IMPACT, WHICH WAY ARE THEY GOING, AND HOW FAST?

LET'S VIEW IT FROM ABOVE:



EASTWARD MOMENTUM $= 200 \frac{M \cdot KG}{SEC}$

NORTHWARD MOMENTUM $= 240 \frac{M \cdot KG}{SEC}$

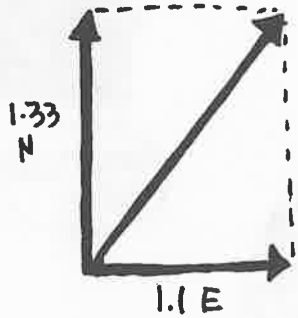
TOTAL MASS $= 180$ kg

SO:

FINAL EASTWARD VELOCITY $= \frac{200}{180} = 1.1$ M/SEC

FINAL NORTHWARD VELOCITY $= \frac{240}{180} = 1.33$ M/SEC

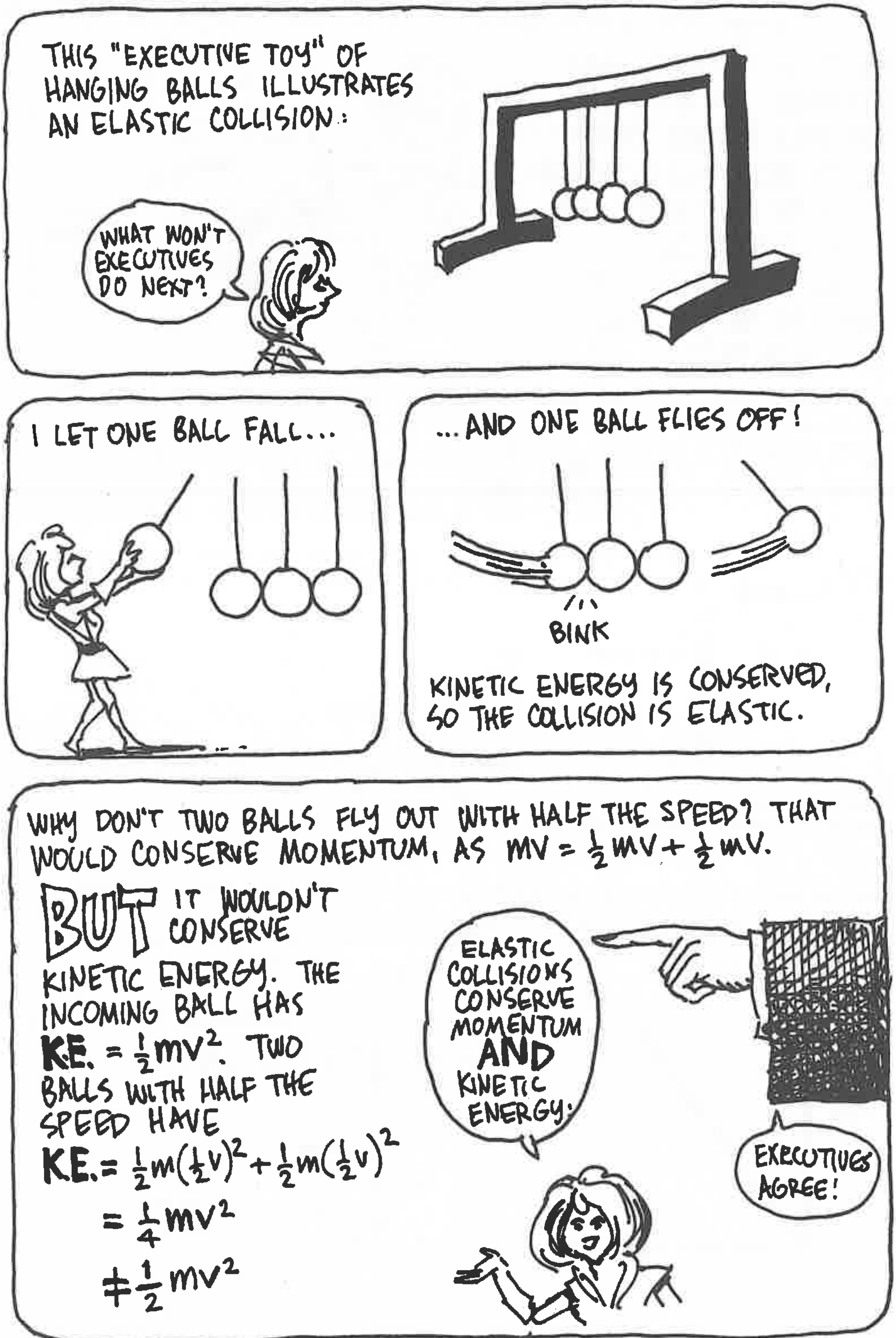
THE FINAL DIRECTION IS THE VECTOR SUM:



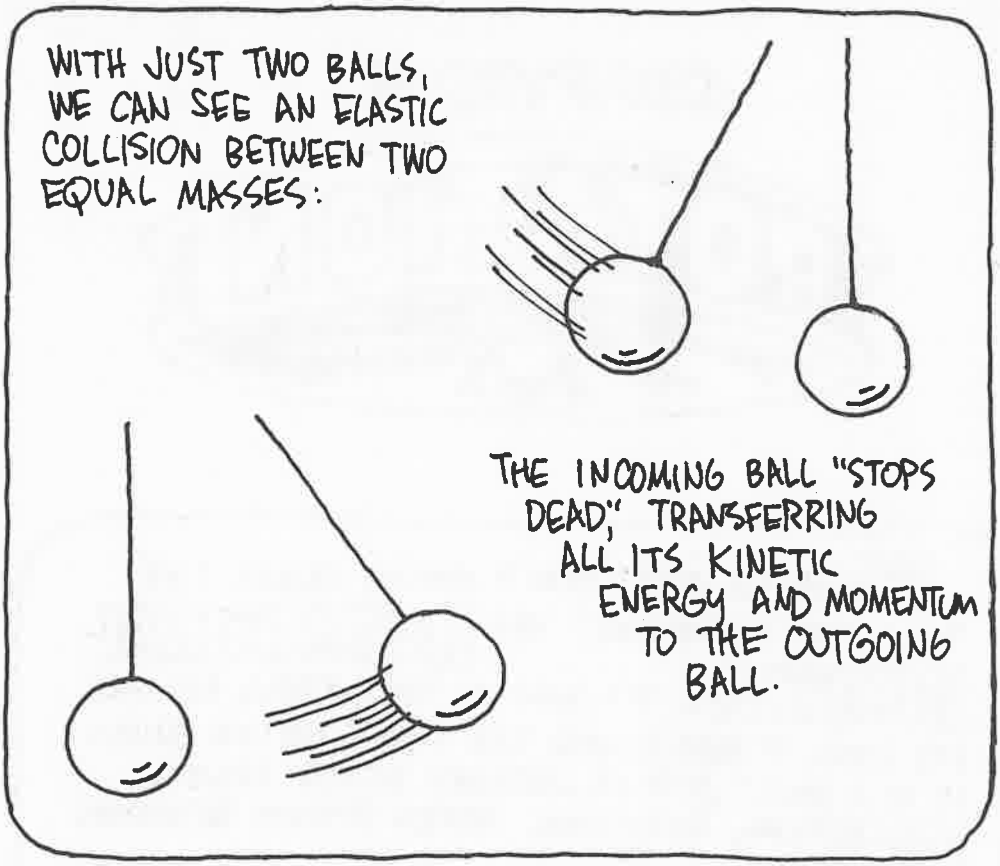
THE LENGTH OF THIS ARROW IS THE FINAL SPEED:

$$V_F = \sqrt{(1.1)^2 + (1.33)^2}$$

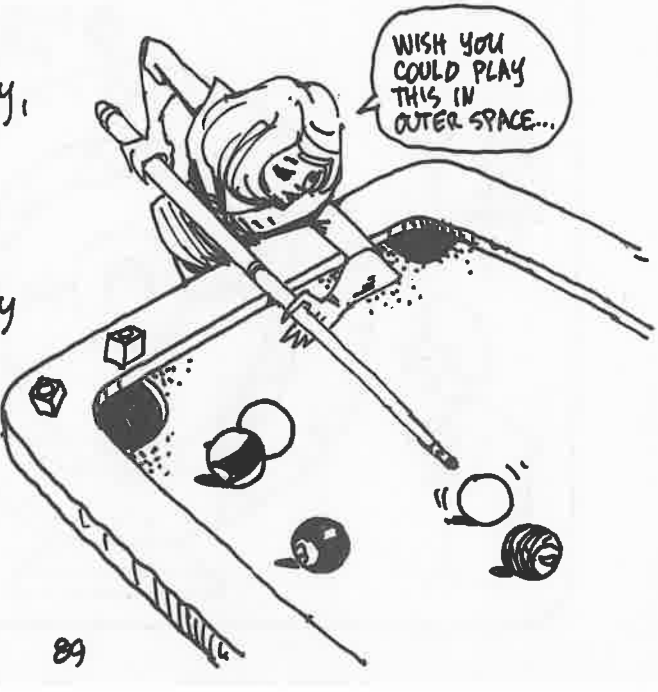
$$= 1.7 \text{ M/SEC.}$$



THIS "EXECUTIVE TOY" OF HANGING BALLS ILLUSTRATES AN ELASTIC COLLISION:
WHAT WON'T EXECUTIVES DO NEXT?
I LET ONE BALL FALL...
...AND ONE BALL FLIES OFF!
BINK
KINETIC ENERGY IS CONSERVED, SO THE COLLISION IS ELASTIC.
WHY DON'T TWO BALLS FLY OUT WITH HALF THE SPEED? THAT WOULD CONSERVE MOMENTUM, AS $mv = \frac{1}{2}mv + \frac{1}{2}mv$.
BUT IT WOULDN'T CONSERVE KINETIC ENERGY. THE INCOMING BALL HAS $K.E. = \frac{1}{2}mv^2$. TWO BALLS WITH HALF THE SPEED HAVE
$K.E. = \frac{1}{2}m(\frac{1}{2}v)^2 + \frac{1}{2}m(\frac{1}{2}v)^2$
$= \frac{1}{4}mv^2$
$\neq \frac{1}{2}mv^2$
ELASTIC COLLISIONS CONSERVE MOMENTUM AND KINETIC ENERGY.
EXECUTIVES AGREE!



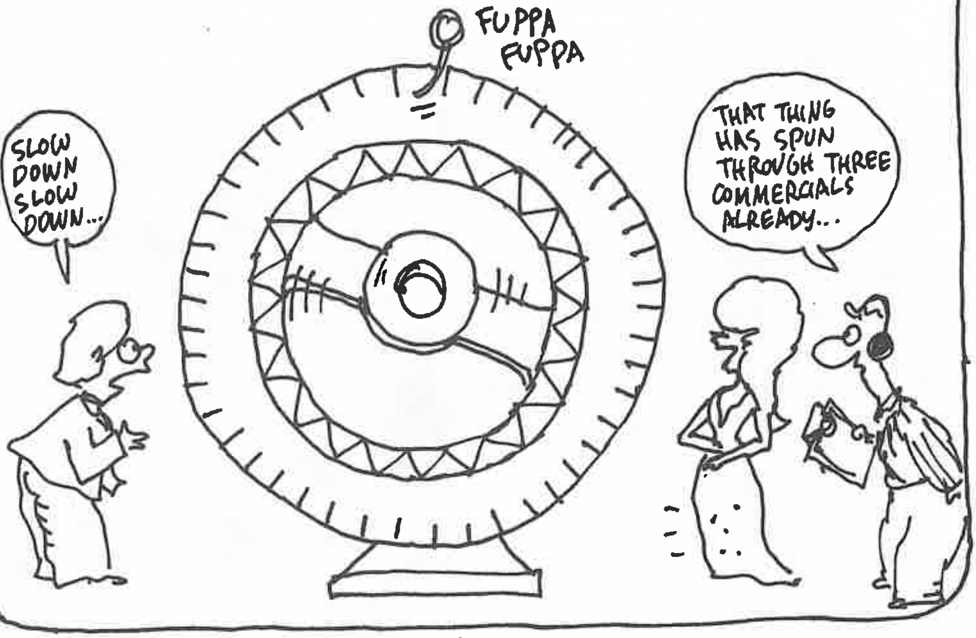
YOU SEE THE SAME SITUATION, APPROXIMATELY, IN THE HEAD-ON COLLISION OF BILLIARD BALLS — BUT WITH BILLIARD BALLS, SOME OF THE KINETIC ENERGY IS IN THE BALL'S **ROTATION**, WHICH BRINGS US TO THE NEXT SECTION...



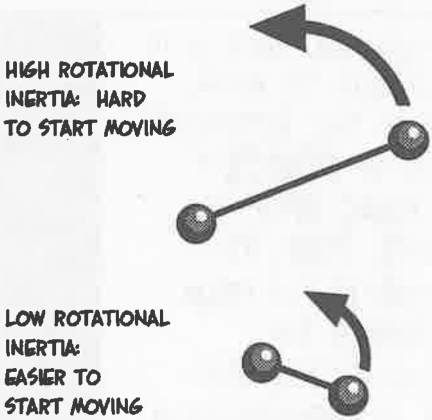
CHAPTER 11

ROTATION

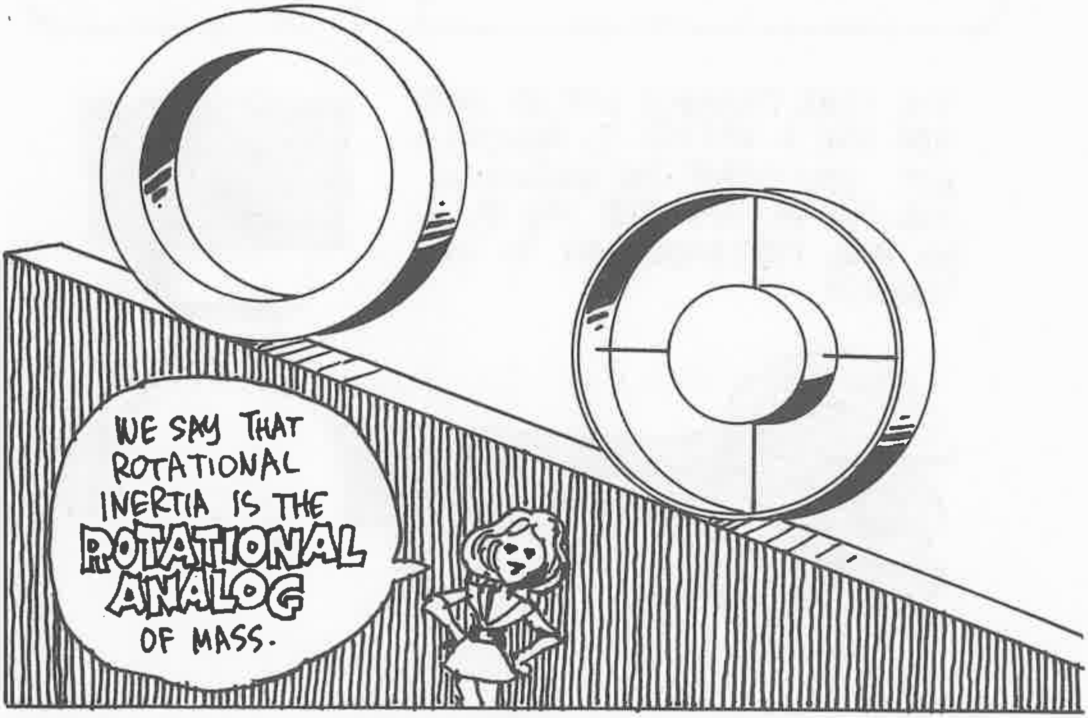
WE ARE ALL AWARE THAT A MASSIVE OBJECT, LIKE THIS "WHEEL OF FORTUNE," HAS **ROTATIONAL INERTIA**. IT'S HARD TO START MOVING, AND ONCE IT'S GOING, IT RUNS A LONG TIME BEFORE FRICTION BRINGS IT TO A HALT. JUST AS ORDINARY INERTIA RESISTS ACCELERATIONS, ROTATIONAL INERTIA RESISTS ROTATIONAL ACCELERATION.

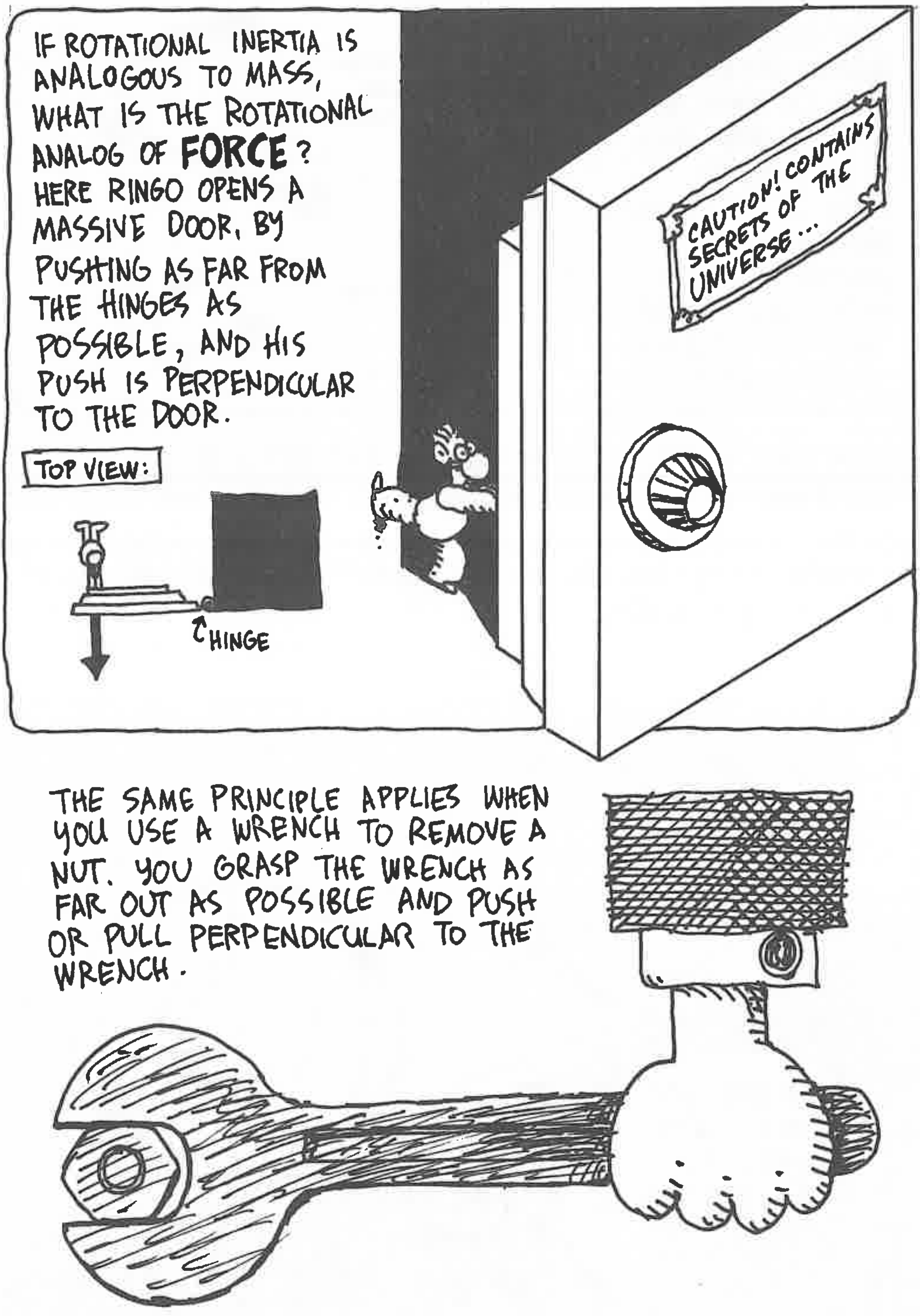


DID YOU REALIZE THAT ROTATIONAL INERTIA DEPENDS NOT ONLY ON MASS, BUT ALSO ON HOW MASS IS DISTRIBUTED? MASS ON THE OUTSIDE, AWAY FROM THE CENTER, HAS MORE ROTATIONAL INERTIA THAN MASS CLOSER TO THE CENTER!



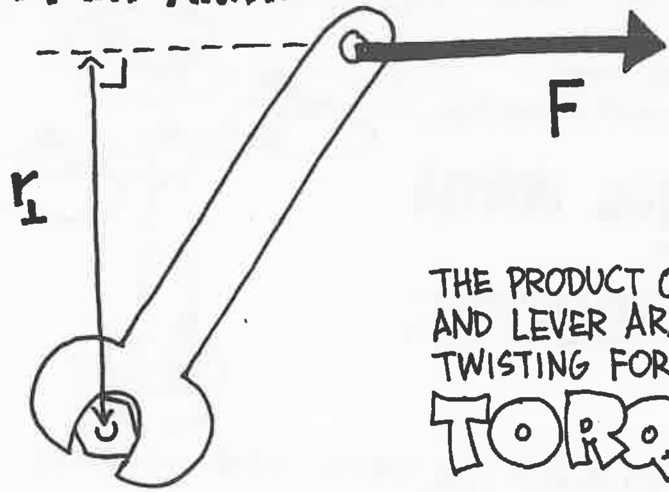
LET'S RACE A "RIM-LOADED" WHEEL AGAINST A MASS-CENTERED WHEEL DOWN AN INCLINED PLANE. THE MASS-CENTERED WHEEL QUICKLY TAKES THE LEAD, BECAUSE IT IS EASIER TO GET ROTATING THAN THE RIM-LOADED WHEEL.




IF ROTATIONAL INERTIA IS ANALOGOUS TO MASS, WHAT IS THE ROTATIONAL ANALOG OF FORCE? HERE RINGO OPENS A MASSIVE DOOR, BY PUSHING AS FAR FROM THE HINGES AS POSSIBLE, AND HIS PUSH IS PERPENDICULAR TO THE DOOR.
TOP VIEW:
HINGE
CAUTION! CONTAINS SECRETS OF THE UNIVERSE...
THE SAME PRINCIPLE APPLIES WHEN YOU USE A WRENCH TO REMOVE A NUT. YOU GRASP THE WRENCH AS FAR OUT AS POSSIBLE AND PUSH OR PULL PERPENDICULAR TO THE WRENCH.

WE CALL $r_\perp$ ("R-PERP"), THE PERPENDICULAR DISTANCE FROM THE PIVOT POINT TO THE LINE OF FORCE, THE

LEVER ARM.



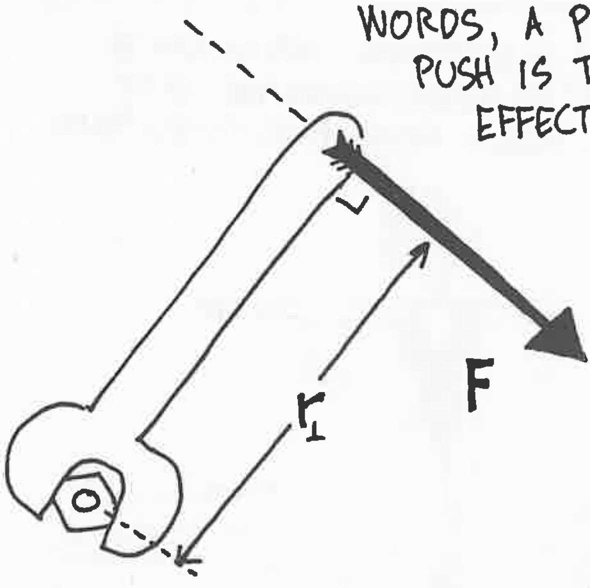
THE PRODUCT OF FORCE AND LEVER ARM IS THE TWISTING FORCE, OR

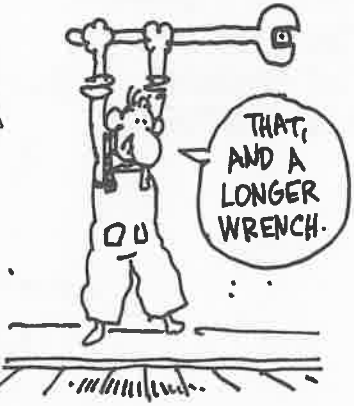
TORQUE.

$$\text{Torque} = F \cdot r_\perp .$$

TORQUE IS THE ROTATIONAL ANALOG OF FORCE.

NOTE HOW MAKING **F** PERPENDICULAR TO THE RADIUS (THE WRENCH) MAXIMIZES $r_\perp$. IN OTHER WORDS, A PERPENDICULAR PUSH IS THE MOST EFFECTIVE PUSH!





OUR FINAL ROTATIONAL ANALOG IS

ANGULAR MOMENTUM.

BY ANALOGY WITH LINEAR MOMENTUM (MASS TIMES VELOCITY), ANGULAR MOMENTUM IS DEFINED AS

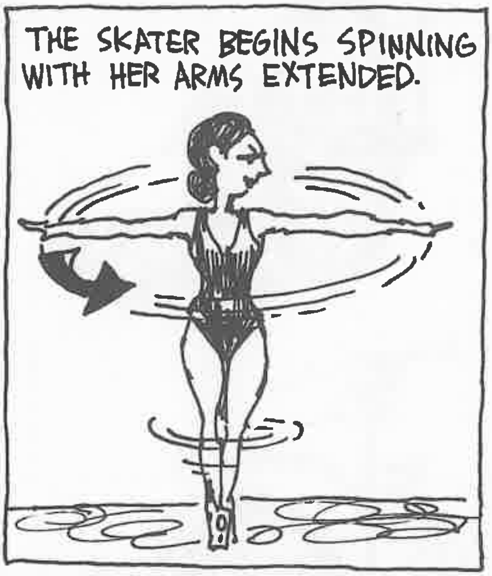
ROTATIONAL INERTIA × ANGULAR VELOCITY.

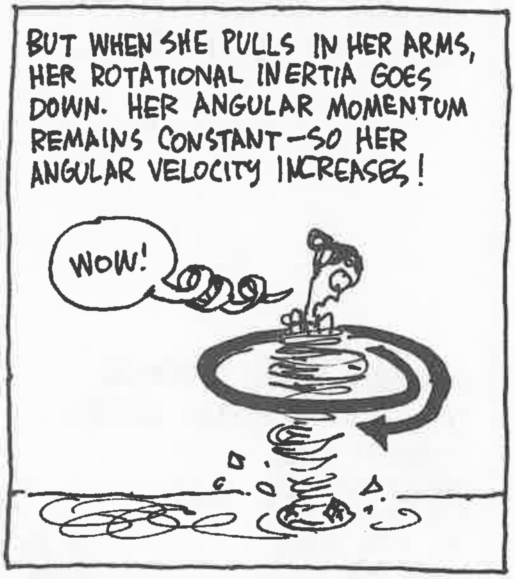
(ANGULAR VELOCITY IS JUST THE TURNING RATE. IT CAN BE EXPRESSED IN REVOLUTIONS PER SECOND.)

UNLIKE MASS, THE AMOUNT OF ROTATIONAL INERTIA CAN BE CHANGED "IN MID-FLIGHT" BY REARRANGING THE MASS. THIS MAKES ROTATIONAL MOTION MORE COMPLICATED THAN LINEAR MOTION.

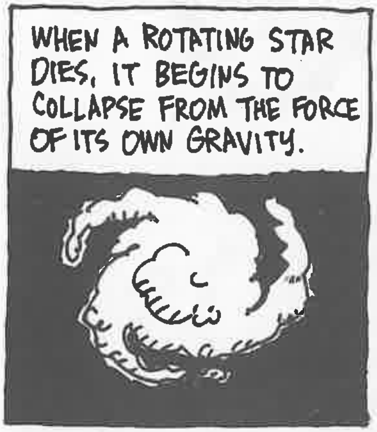
TAKE, FOR EXAMPLE, THE CASE OF THE SPINNING ICE SKATER...

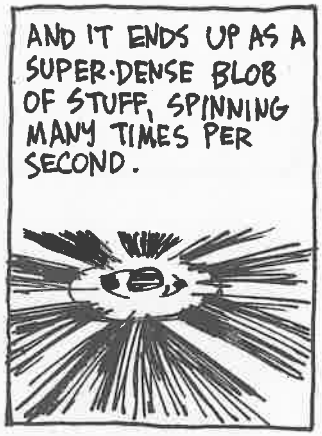
REMEMBER THAT MOMENTUM IS CONSERVED IN THE ABSENCE OF EXTERNAL FORCES. LIKEWISE, **ANGULAR** MOMENTUM IS CONSERVED IN THE ABSENCE OF EXTERNAL **TORQUES.**



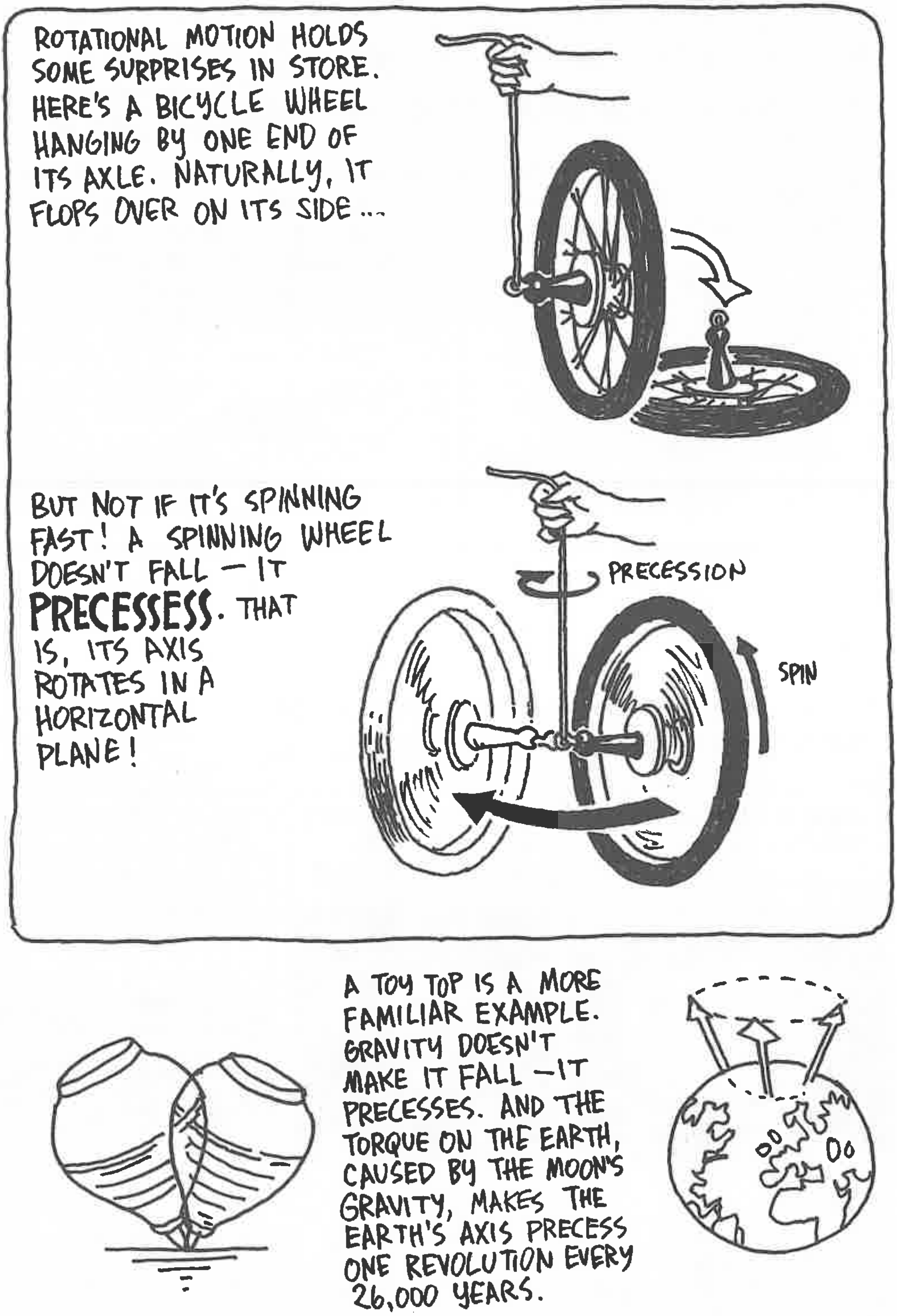


IN THIS RESPECT, AN ICE SKATER RESEMBLES A COLLAPSING STAR. THEY BOTH CONSERVE ANGULAR MOMENTUM!

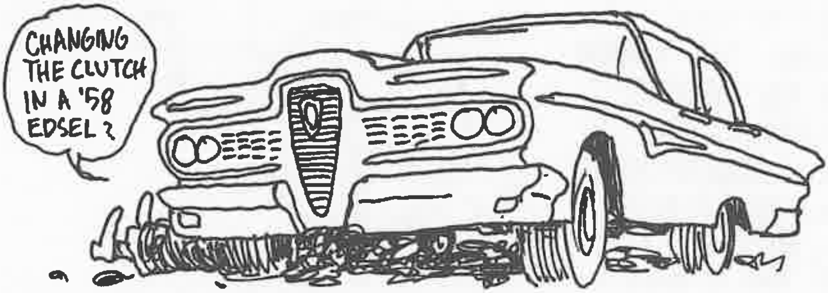




LARGE ROTATIONAL INERTIA × SMALL SPIN RATE
=
SMALL ROTATIONAL INERTIA × LARGE SPIN RATE

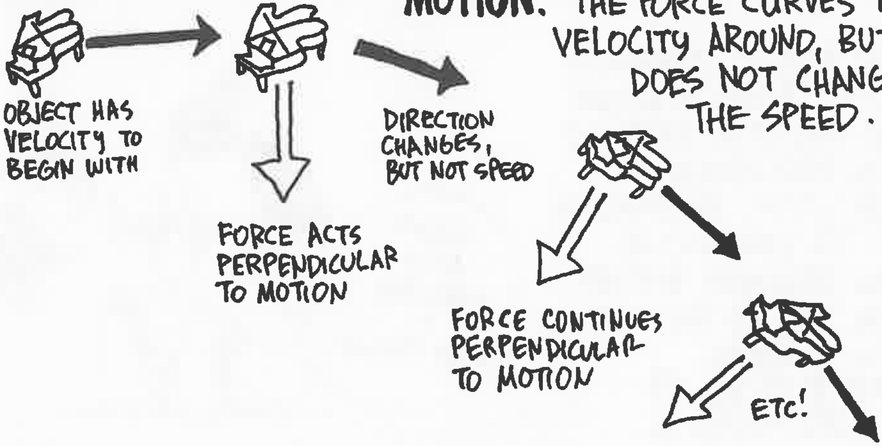

ROTATIONAL MOTION HOLDS SOME SURPRISES IN STORE. HERE'S A BICYCLE WHEEL HANGING BY ONE END OF ITS AXLE. NATURALLY, IT FLOPS OVER ON ITS SIDE...
BUT NOT IF IT'S SPINNING FAST! A SPINNING WHEEL DOESN'T FALL — IT PRECESSESS. THAT IS, ITS AXIS ROTATES IN A HORIZONTAL PLANE!
PRECESSION
SPIN
A TOY TOP IS A MORE FAMILIAR EXAMPLE. GRAVITY DOESN'T MAKE IT FALL —IT PRECESSES. AND THE TORQUE ON THE EARTH, CAUSED BY THE MOON'S GRAVITY, MAKES THE EARTH'S AXIS PRECESS ONE REVOLUTION EVERY 26,000 YEARS.

NOW LET'S PUT OUR MECHANICS KNOWLEDGE TO THE ULTIMATE TEST:

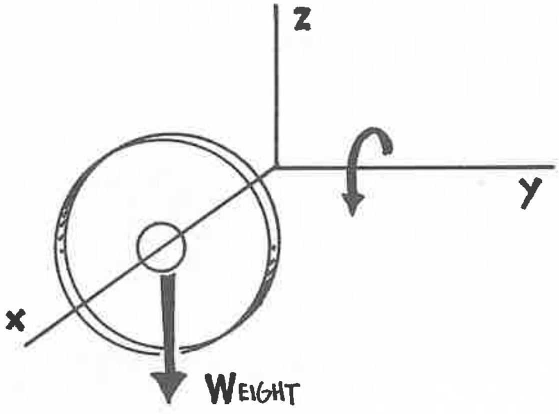


NO... LET'S SEE IF WE CAN UNDERSTAND PRECESSION.... BUT FIRST, AN OBSERVATION ABOUT LINEAR MOTION: SUPPOSE AN OBJECT IS AT REST, AND A FORCE ACTS ON IT. THEN THE OBJECT STARTS TO ACCELERATE IN THE DIRECTION OF THE FORCE.

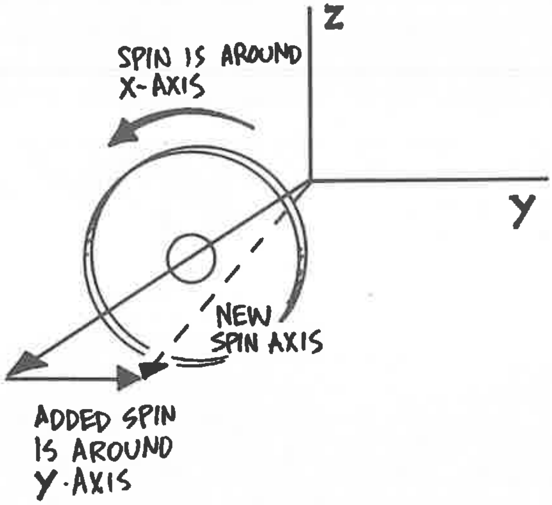
HOWEVER, IF THE OBJECT IS ALREADY MOVING, AND THE FORCE ALWAYS ACTS AT RIGHT ANGLES TO THE MOTION, WE GET **UNIFORM CIRCULAR MOTION.** THE FORCE CURVES THE VELOCITY AROUND, BUT DOES NOT CHANGE THE SPEED.



SOMETHING SIMILAR HAPPENS WITH ROTATION: WHEN THE WHEEL IS NOT SPINNING, THE TORQUE FROM THE WEIGHT ACCELERATES THE WHEEL ANGULARLY AROUND THE TORQUE AXIS, IN THIS CASE THE **y**-AXIS.

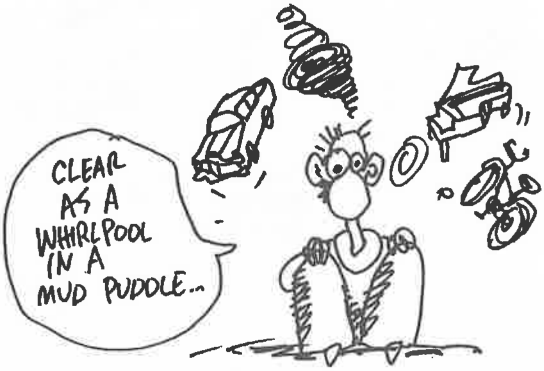


BUT IF THE WHEEL IS SPINNING, IT ALREADY HAS ANGULAR MOMENTUM AROUND THE **X**-AXIS. THE TORQUE ADDS SOME SPIN AROUND THE **y**-AXIS, PERPENDICULAR TO THE ORIGINAL SPIN. THE RESULTING SPIN AXIS IS TURNED A LITTLE IN THE **x-y** PLANE.



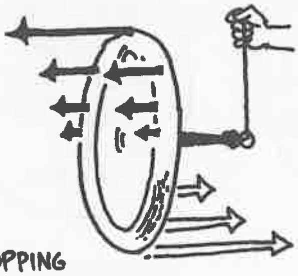
THE TORQUE CONTINUES TO ACT PERPENDICULAR TO THE SPIN AXIS, SO THE PRECESSION CONTINUES. ANALOGOUSLY TO THE LINEAR CASE, THE DIRECTION, BUT NOT THE SIZE, OF THE SPIN HAS CHANGED.

CLEAR?



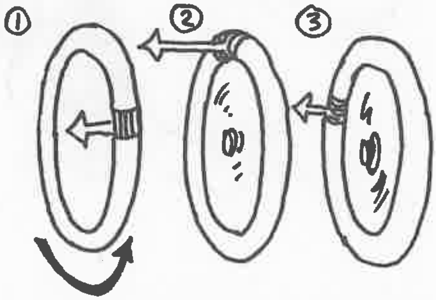
THE ARGUMENT ON THE LAST PAGE WAS BASED ON THE CONCEPTS OF TORQUE AND ANGULAR MOMENTUM. BUT THESE CONCEPTS ARE ULTIMATELY BASED ON NEWTON'S SECOND LAW, F = ma. LET'S SEE IF WE CAN UNDERSTAND PRECESSION JUST FROM F = ma.

FIRST: THE TORQUE EXERTED BY GRAVITY TENDS TO MAKE THE WHEEL FLOP OVER. SO THERE'S AN OUTWARD FORCE ON THE TOP HALF OF THE WHEEL, AND AN INWARD FORCE ON THE BOTTOM HALF.

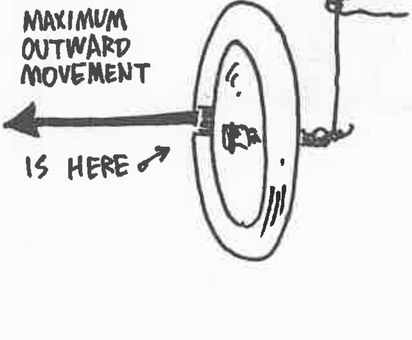


FLOPPING FORCES ON WHEEL

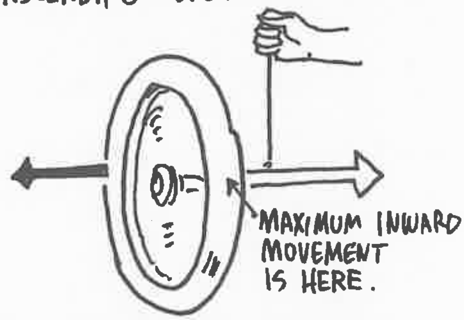
NOW LOOK AT A SMALL PIECE OF THE WHEEL AS IT SPINS. AS IT PASSES THROUGH THE UPPER HALF, IT EXPERIENCES CONTINUAL OUTWARD FORCE.



THEREFORE, IT ACCELERATES **OUTWARD**, REACHING MAXIMUM OUTWARD VELOCITY WHEN IT IS AT THE SIDEWAYS POSITION, ABOUT TO ENTER THE WHEEL'S BOTTOM HALF.

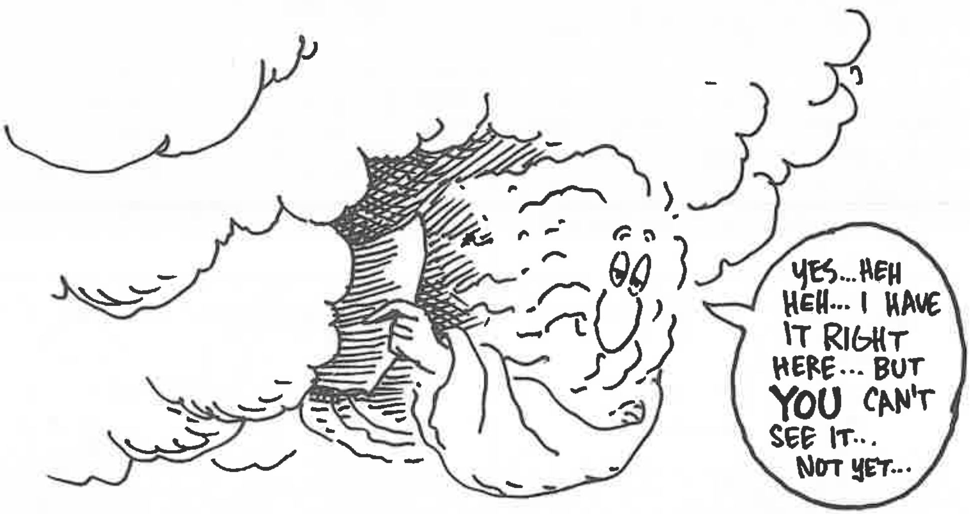


SIMILARLY, EACH PIECE OF THE WHEEL HAS MAXIMUM **INWARD** VELOCITY AT THE "HORIZONTAL-ASCENDING" SPOT.



SO – AS YOU SEE, THE WHEEL PRECESSES INSTEAD OF FLOPPING!

WELL, I SPENT SO MUCH TIME EXPLAINING PRECESSION TO YOU IN ORDER TO SHOW HOW COMPLICATED THINGS CAN GET, JUST STARTING FROM THAT SIMPLE EQUATION **F = ma**. PHYSICS IS AMAZING THAT WAY.... WHO KNOWS? MAYBE WE **WILL** REDUCE THE PHYSICS OF THE ENTIRE UNIVERSE TO A PAGE FULL OF EQUATIONS !!




NOW WHERE DID I PUT THAT LIST?