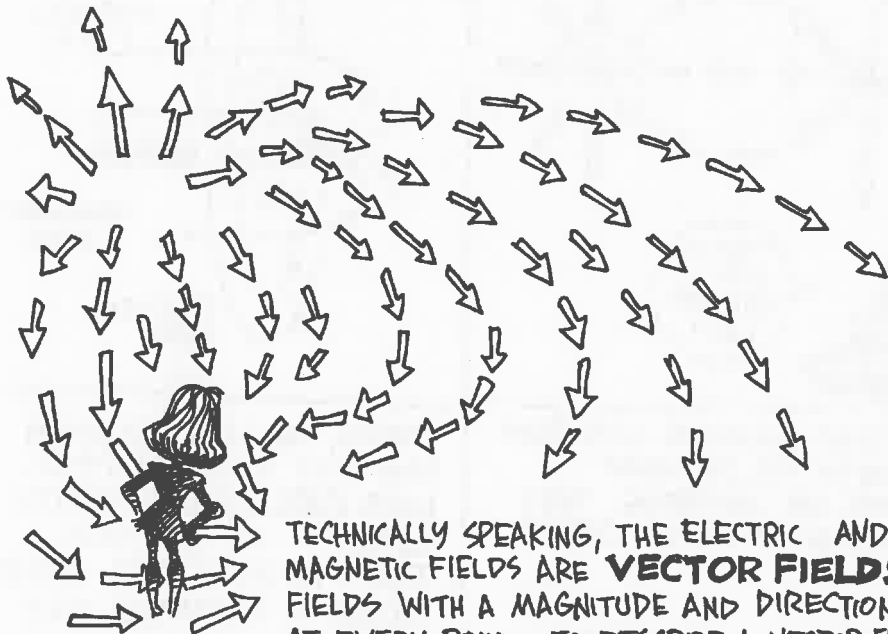


◊ CHAPTER 23 ◊

# MAXWELL'S EQUATIONS AND LIGHT (AND HOW!)



TECHNICALLY SPEAKING, THE ELECTRIC AND MAGNETIC FIELDS ARE **VECTOR FIELDS** - FIELDS WITH A MAGNITUDE AND DIRECTION AT EVERY POINT. TO DESCRIBE A VECTOR FIELD, YOU MUST SPECIFY HOW THE FIELD SPREADS OUT, OR **DIVERGES**, AND HOW IT CIRCLES AROUND, OR **CURLS**. (DIVERGENCE AND CURL ARE MATHEMATICAL TERMS.)

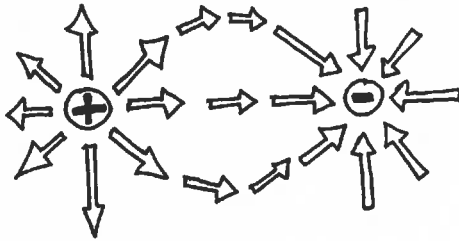
IN 1873, JAMES CLERK

## MAXWELL

WROTE DOWN FOUR EQUATIONS WHICH SPECIFY THE CURL AND DIVERGENCE OF THE ELECTRIC AND MAGNETIC FIELDS.

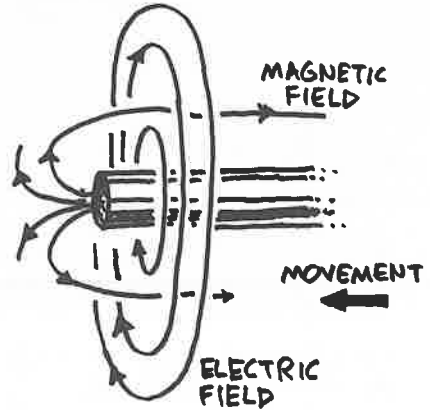


MAXWELL'S FIRST EQUATION IS GAUSS'S LAW. IT SAYS THAT ELECTRIC FIELD LINES **DIVERGE** FROM POSITIVE CHARGES AND **CONVERGE** TO NEGATIVE CHARGES.

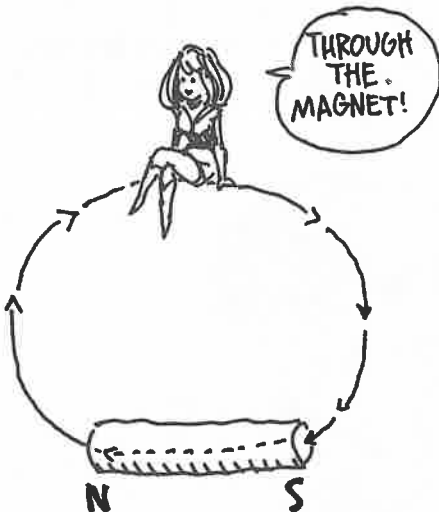


THIS IS ALSO COULOMB'S LAW!

THE SECOND EQUATION IS FARADAY'S LAW: ELECTRIC FIELD LINES **CURL** AROUND CHANGING MAGNETIC FIELDS. CHANGING MAGNETIC FIELDS INDUCE ELECTRIC FIELDS.

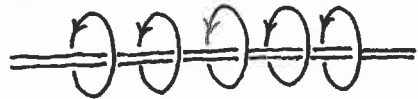


THE THIRD EQUATION SAYS THAT MAGNETIC FIELDS NEVER DIVERGE OR CONVERGE. THEY ALWAYS GO IN CLOSED CURVES.



THROUGH THE MAGNET!

FINALLY, THE LAST EQUATION SAYS THAT MAGNETIC FIELD LINES **CURL** AROUND ELECTRIC CURRENTS. WE HAVE SEEN THAT A MAGNETIC FIELD CIRCLES AROUND A CONDUCTING WIRE.



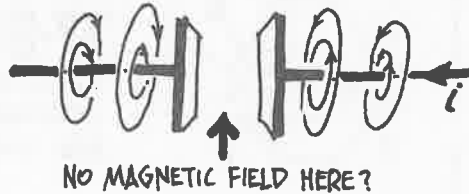
...AND HERE MAXWELL HAD A CRITICAL BRAINSTORM! (AN ELECTRICAL STORM, OF COURSE!)



AS YOU SEE, THE EQUATIONS EXPRESS LAWS THAT CAME TO MAXWELL FROM OTHER SOURCES. BUT MAXWELL'S GENIUS WAS TO SEE THAT LAW #4 WAS INCOMPLETE.



CONSIDER A CAPACITOR BEING CHARGED. AS THE CURRENT FLOWS TO THE CAPACITOR PLATES, A MAGNETIC FIELD RINGS THE WIRE. BUT WHAT ABOUT BETWEEN THE PLATES?



DOES THE FIELD STOP ABRUPTLY BETWEEN THE PLATES, WHERE THE CURRENT STOPS? MAXWELL SAID —



MAXWELL FELT THAT NATURE DISLIKES DISCONTINUITIES. ALSO, HE REASONED THAT IF CHANGING MAGNETIC FIELDS INDUCE ELECTRIC FIELDS (FARADAY), THEN, SYMMETRICALLY, CHANGING ELECTRIC FIELDS MIGHT INDUCE MAGNETIC FIELDS. THERE WAS NO EVIDENCE FOR THIS, OF COURSE, BUT...



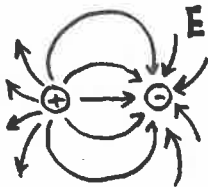
THUS, MAXWELL ADDED AN EXTRA TERM TO HIS FOURTH EQUATION, SAYING THAT MAGNETIC FIELDS ALSO **CURL** AROUND CHANGING ELECTRIC FIELDS. THIS TERM GENERATES A MAGNETIC FIELD BETWEEN THE CAPACITOR PLATES AS THE ELECTRIC FIELD BUILDS UP.



SOME YEARS LATER, THIS MAGNETIC FIELD WAS DETECTED.

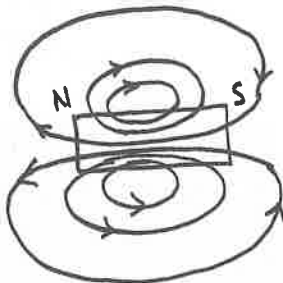


SO, WITHOUT FURTHER ADD, HERE ARE  
**MAXWELL'S EQUATIONS**  
 IN FULL-BLOWN MATHEMATICAL NOTATION  
 TO THRILL AND INTIMIDATE YOU!



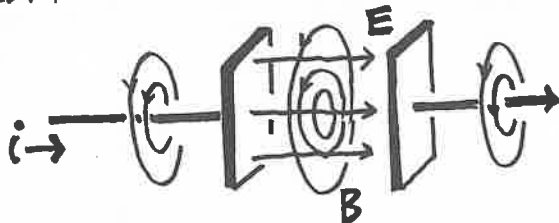
$\nabla \cdot \mathbf{E} = 4\pi\rho$  ( $\rho$ , GREEK "RHO" = CHARGE DENSITY;  $\mathbf{E}$  = ELECTRIC FIELD) SAYS  $\mathbf{E}$  DIVERGES OUTWARD FROM PLUS CHARGES AND INWARD TO MINUS CHARGES.

$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{d\mathbf{B}}{dt}$  ( $\mathbf{B}$  = MAGNETIC FIELD)  
 SAYS  $\mathbf{E}$  CURLS AROUND CHANGING  $\mathbf{B}$  FIELDS.  
 ( $c$  = SPEED OF LIGHT)



$\nabla \cdot \mathbf{B} = 0$  SAYS  $\mathbf{B}$  NEVER DIVERGES, ALWAYS LOOPS AROUND.

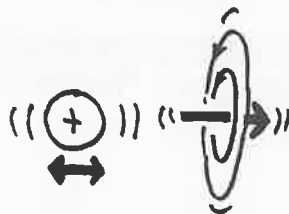
$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{d\mathbf{E}}{dt}$   
 SAYS  $\mathbf{B}$  CURLS AROUND CURRENTS  
 ( $\mathbf{J}$  = CURRENT DENSITY) AND  
 CHANGING  $\mathbf{E}$  FIELDS.



THAT ONE LITTLE TERM  
ADDED TO MAXWELL'S FOURTH  
EQUATION HAD AN UNEXPECTED  
PAYOFF — AND A BIG ONE.

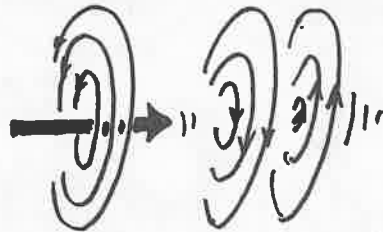


IMAGINE A SINGLE ELECTRIC CHARGE BEING **VIBRATED**:



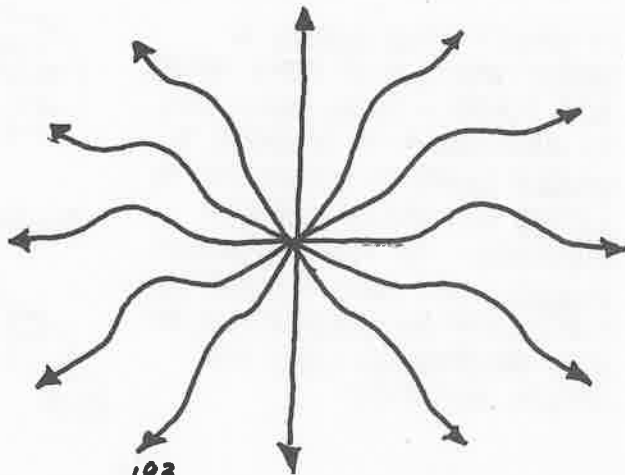
IN THE SPACE NEAR THE  
VIBRATING CHARGE, THE  
CHARGE'S ELECTRIC FIELD IS  
CHANGING, SO IT INDUCES A  
MAGNETIC FIELD CURLING  
AROUND IT.

BUT THE MAGNETIC FIELD  
IS ALSO CHANGING — SO  
IT INDUCES MORE ELECTRIC  
FIELD, WHICH INDUCES  
MORE MAGNETIC FIELD...



**ETC.!**

THE RESULT IS A  
**WAVE**  
OF FIELDS RIPPLING  
OUT FROM THE  
VIBRATING  
CHARGE — AT THE  
SPEED OF LIGHT,  
ACCORDING TO  
MAXWELL'S  
CALCULATIONS!





MAXWELL HAD A FLASH OF ILLUMINATION! **LIGHT** ITSELF, HE HYPOTHEZIZED, IS SUCH AN **ELECTRO-MAGNETIC WAVE.**

SOON AFTERWARD, HERTZ AND OTHERS DID INDEED PRODUCE LONG ELECTROMAGNETIC WAVES FROM WIGGLING CHARGES— AND DETECTED THEM AT A DISTANCE FROM THE SOURCE!



IT WASN'T LONG BEFORE A WHOLE SPECTRUM OF THESE WAVES WAS FOUND— FROM RADIO WAVES TO MICROWAVES TO INFRARED TO VISIBLE LIGHT TO ULTRAVIOLET TO X-RAYS AND NUCLEAR GAMMA RADIATION. IN FOUR EQUATIONS, MAXWELL NOT ONLY SUMMARIZED ELECTRICITY AND MAGNETISM, BUT ALSO ENCOMPASSED LIGHT AND OPTICS! NOT BAD!!

