1 Motion

1-1 Speed, Velocity, and Acceleration

Speed vs. Velocity

Vocabulary **Distance:** How far something travels.

Vocabulary **Displacement:** How far something travels in a given direction.

Notice that these two terms are very similar. **Distance** is an example of what we call a *scalar* quantity. In other words, it has magnitude, but no direction. **Displacement** is an example of a *vector* quantity because it has both magnitude and direction.

The SI (Système International) unit for distance and displacement is the **meter (m).**

Displacements smaller than a meter may be expressed in units of centimeters (cm) or millimeters (mm). Displacements much larger than a meter may be expressed in units of kilometers (km). See Appendix A for the meanings of these and other common prefixes.

Vocabulary **Speed:** How fast something is moving.

$$\textbf{average speed} = \frac{\textbf{distance traveled}}{\textbf{elapsed time}} \quad \text{or} \quad v_{av} = \frac{d}{\Delta t}$$

Vocabulary **Velocity:** How fast something is moving in a given direction.

$$\textbf{average velocity} = \frac{\textbf{displacement}}{\textbf{elapsed time}} \quad \text{or} \quad \mathbf{v_{av}} = \frac{\mathbf{\Delta d}}{\Delta t} = \frac{\mathbf{d_f} - \mathbf{d_o}}{t_f - t_o}$$

where d_f and t_f are the final position and time respectively, and d_o and t_o are the initial position and time. The symbol "Δ" (delta) means "change" so Δd is the change in position, or the displacement, while Δt is the change in time.

In this book all vector quantities will be introduced in an equation with **bold type** while all scalar quantities will be introduced in an equation in regular type. Note that speed is a scalar quantity while velocity is a vector quantity.

The SI unit for both speed and velocity is the **meter per second (m/s).**

When traveling in any moving vehicle, you rarely maintain the same velocity throughout an entire trip. If you did, you would travel at a constant speed in a straight line. Instead, speed and direction usually vary during your time of travel.

If you begin and end at the same location but you travel for a great distance in getting there (for example, when you travel in a circle), you have a measurable average speed. However, since your total displacement for such a trip is zero, your average velocity is also zero. In this chapter, both average speed and average velocity will be written as v_{av}. The "av" subscript will be dropped in later chapters.

Acceleration

Vocabulary **Acceleration:** The rate at which the velocity changes during a given amount of time.

$$\textbf{acceleration} = \frac{\textbf{change in velocity}}{\textbf{elapsed time}} \quad \text{or} \quad a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

where the terms v_f and v_o mean final velocity and initial velocity, respectively.

The SI unit for acceleration is the **meter per second squared (m/s^2).**

If the final velocity of a moving object is smaller than its initial velocity, the object must be slowing down. A slowing object is sometimes said to have *negative acceleration* because the magnitude of the acceleration is preceded by a negative sign.

Solved Examples

Example 1: Benjamin watches a thunderstorm from his apartment window. He sees the flash of a lightning bolt and begins counting the seconds until he hears the clap of thunder 10. s later. Assume that the speed of sound in air is 340 m/s. How far away was the lightning bolt a) in m? b) in km? (Note: The speed of light, 3.0×10^8 m/s, is considerably faster than the speed of sound. That is why you see the lightning flash so much earlier than you hear the clap of thunder. In actuality, the lightning and thunder clap occur almost simultaneously.)

a. *Given:* $v_{av} = 340$ m/s
$\Delta t = 10.0$ s

Unknown: $\Delta d = ?$
Original equation: $v_{av} = \dfrac{\Delta d}{\Delta t}$

Solve: $\Delta d = v_{av}\Delta t = (340 \text{ m/s})(10.\text{ s}) = \mathbf{3400\ m}$

b. For numbers this large you may wish to express the final answer in km rather than in m. Because "kilo" means 1000, then 1.000 km = 1000. m.

$$3400 \text{ m} \frac{(1.000 \text{ km})}{1000. \text{ m}} = \mathbf{3.4 \text{ km}}$$

The lightning bolt is 3.4 km away, which is just a little over two miles for those of you who think in English units!

Example 2: On May 28, 2000, Juan Montoya became the first Colombian citizen to win the Indianapolis 500. Montoya completed the race in a time of 2.98 h. What was Montoya's average speed during the 500.-mi race? (Note: Generally the unit "miles" is not used in physics exercises. However, the Indianapolis 500 is a race that is measured in miles, so the mile is appropriate here. Don't forget, the SI unit for distance is the meter.)

Given: $d = 500.$ mi
$\Delta t = 2.98$ h

Unknown: $v_{av} = ?$
Original equation: $\Delta t = \dfrac{\Delta d}{v}$

Solve: $\Delta t = \dfrac{\Delta d}{v} = \dfrac{500. \text{ mi}}{2.98 \text{ h}} = 168 \text{ mi/h}$

Example 3: The slowest animal ever discovered was a crab found in the Red Sea. It traveled with an average speed of 5.70 km/y. How long would it take this crab to travel 100. km?

Given: $\Delta d = 100.$ km
$v_{av} = 5.70$ km/y

Unknown: $\Delta t = ?$
Original equation: $\Delta t = \dfrac{\Delta d}{v_{av}}$

Solve: $\Delta t = \dfrac{\Delta d}{v_{av}} = \dfrac{100. \text{ km}}{5.70 \text{ km/y}} = \mathbf{17.5 \text{ y}}$ A long time!

Example 4: Tiffany, who is opening in a new Broadway show, has some limo trouble in the city. With only 8.0 minutes until curtain time, she hails a cab and they speed off to the theater down a 1000.-m-long one-way street at a speed of 25 m/s. At the end of the street the cab driver waits at a traffic light for 1.5 min and then turns north onto a 1700.-m.-long traffic-filled avenue on which he is able to travel at a speed of only 10.0 m/s. Finally, this brings them to the theater. a) Does Tiffany arrive before the theater lights dim? b) Draw a distance vs. time graph of the situation.

Solution: First, break this exercise down into segments and solve each segment independently.

Segment 1: (one-way street)

Given: $\Delta d = 1000.$ m
$v_{av} = 25$ m/s

Unknown: $\Delta t = ?$
Original equation: $v_{av} = \dfrac{\Delta d}{\Delta t}$

Solve: $\Delta t = \dfrac{\Delta d}{v_{av}} = \dfrac{1000. \text{ m}}{25 \text{ m/s}} = 40. \text{ s}$

Segment 2: (traffic light)

Given: $\Delta t = 1.5$ min $\qquad (1.5 \text{ min})\dfrac{(60.\text{ s})}{(1.0 \text{ min})} = 90.\text{ s}$

Segment 3: (traffic-filled avenue)

Given: $\Delta d = 1700.$ m
$v_{av} = 10.0$ m/s

Unknown: $\Delta t = ?$
Original equation: $v_{av} = \dfrac{\Delta d}{\Delta t}$

Solve: $\Delta t = \dfrac{\Delta d}{v_{av}} = \dfrac{1700.\text{ m}}{10.0\text{ m/s}} = 170.\text{ s}$

total time $= 40.\text{ s} + 90.\text{ s} + 170.\text{ s} = 300.\text{ s} \qquad (300.\text{ s})\dfrac{(1.0\text{ min})}{(60.\text{ s})} = \mathbf{5.0\ min}$

Yes, she not only makes it to the show in time, but she even has 3.0 minutes to spare to put on her costume and make-up.

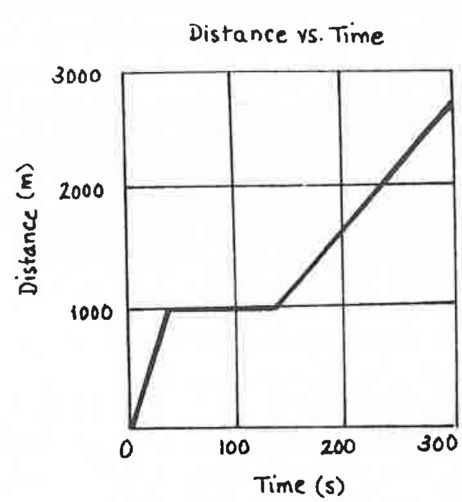
b. The motion of the cab can be described by the following graph.

In Segment 1, the distance of 1000. m was covered in a fairly short amount of time, which means that the cab was traveling quickly. This high speed can be seen as a steep slope on the graph.

In Segment 2, the cab was at rest. Notice that even though the cab did not move, time continued on, resulting in a horizontal line on the graph.

In Segment 3, the distance of 1700. m was covered in a much longer amount of time so the cab was traveling slowly. This low speed is indicated by a slope that is not as steep as that in segment 1.

Remember, all graphs should have titles and the axes should be labeled with the correct units.



Example 5: Grace is driving her sports car at 30 m/s when a ball rolls out into the street in front of her. Grace slams on the brakes and comes to a stop in 3.0 s. What was the acceleration of Grace's car?

Given: $v_o = 30\ \text{m/s}$
$v_f = 0\ \text{m/s}$
$\Delta t = 3.0\ \text{s}$

Unknown: $a = ?$
Original equation: $a = \dfrac{v_f - v_o}{\Delta t}$

Solve: $a = \dfrac{v_f - v_o}{\Delta t} = \dfrac{0\ \text{m/s} - 30\ \text{m/s}}{3.0\ \text{s}} = -10\ \text{m/s}^2$

The negative sign means the car was slowing down.

Practice Exercises

Exercise 1: Hans stands at the rim of the Grand Canyon and yodels down to the bottom. He hears his yodel echo back from the canyon floor 5.20 s later. Assume that the speed of sound in air is 340.0 m/s. How deep is the canyon at this location?

Answer: ____________________

Exercise 2: The world speed record on water was set on October 8, 1978 by Ken Warby of Blowering Dam, Australia. If Ken drove his motorboat a distance of 1000. m in 7.045 s, how fast was his boat moving a) in m/s? b) in mi/h?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 3: According to the World Flying Disk Federation, on April 8, 2000, Jennifer Griffin of Fredericksburg, Virginia threw a Frisbee for a distance of 138.56 m to capture the women's record. If the Frisbee was thrown horizontally with a speed of 13.0 m/s, how long did the Frisbee remain aloft?

Answer: ____________________

Exercise 4: It is now 10:29 a.m., but when the bell rings at 10:30 a.m. Suzette will be late for French class for the third time this week. She must get from one side of the school to the other by hurrying down three different hallways. She runs down the first hallway, a distance of 35.0 m, at a speed of 3.50 m/s. The second hallway is filled with students, and she covers its 48.0-m length at an average speed of 1.20 m/s. The final hallway is empty, and Suzette sprints its 60.0-m length at a speed of 5.00 m/s. a) Does Suzette make it to class on time or does she get detention for being late again? b) Draw a distance vs. time graph of the situation.

Answer: **a.** ____________________

Exercise 5: A jumbo jet taxiing down the runway receives word that it must return to the gate to pick up an important passenger who was late to his connecting flight. The jet is traveling at 45.0 m/s when the pilot receives the message. What is the acceleration of the plane if it takes the pilot 5.00 s to bring the plane to a halt?

Answer: ____________________

Exercise 6: While driving his sports car at 20.0 m/s down a four-lane highway, Eddie comes up behind a slow-moving dump truck and decides to pass it in the left-hand lane. If Eddie can accelerate at 5.00 m/s^2, how long will it take for him to reach a speed of 30.0 m/s?

Answer: ____________________

Exercise 7: Vivian is walking to the hairdresser's at 1.3 m/s when she glances at her watch and realizes that she is going to be late for her appointment. Vivian gradually quickens her pace at a rate of 0.090 m/s^2. a) What is Vivian's speed after 10.0 s? b) At this speed, is Vivian walking, jogging, or running very fast?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 8: A torpedo fired from a submerged submarine is propelled through the water with a speed of 20.00 m/s and explodes upon impact with a target 2000.0 m away. If the sound of the impact is heard 101.4 s after the torpedo was fired, what is the speed of sound in water? (Because the torpedo is held at a constant speed by its propeller, the effect of water resistance can be neglected.)

Answer: ____________________

1-2 Free Fall

Vocabulary **Free Fall:** The movement of an object in response to a gravitational attraction.

When an object is released, it falls toward the earth due to the gravitational attraction the earth provides. As the object falls, it will accelerate at a constant rate of 9.8 m/s^2 regardless of its mass. However, to make calculations more expedient and easier to do without a calculator, this number will be written as $g = 10.0\ m/s^2$ throughout this book.

There are many different ways to solve free fall exercises. The sign convention used may be chosen by you or your teacher. In this book, the downward direction will be positive, and anything falling downward will be written with a positive velocity and position; anything moving upward will be represented with a negative velocity and position. Remember: Gravity *always* acts to pull an object down, so the gravitational acceleration, g, will always be written as a positive number regardless of which direction the object is moving.

The displacement of a falling object in a given amount of time is written as

$$\Delta d = v_o \Delta t + \left(\frac{1}{2}\right) g \Delta t^2$$

The final velocity of a falling object can be represented by the equation

$$v_f^2 = v_o^2 + 2g\Delta d$$

or by the earlier equation, $a = (v_f - v_o)/\Delta t$, rewritten as $v_f = v_o + a\Delta t$, or

$$v_f = v_o + g\Delta t$$

Note that the term "g" in all of these exercises can be written as "a" if you use a constant acceleration other than gravity. Therefore, these equations can be used for objects moving horizontally as well as vertically.

It is common to neglect air resistance in most free fall exercises (including those in this book), although in real life, air resistance is a factor that must be taken into account. This book will also assume that the initial speed of all objects in free fall is zero, unless otherwise specified.

Solved Examples

Example 6: King Kong carries Fay Wray up the 321-m-tall Empire State Building. At the top of the skyscraper, Fay Wray's shoe falls from her foot. How fast will the shoe be moving when it hits the ground?

Given: $v_o = 0$ m/s
$g = 10.0$ m/s^2
$\Delta d = 321$ m

Unknown: $v_f = ?$
Original equation: $v_f^2 = v_o^2 + 2g\Delta d$

Solve: $v_f = \sqrt{v_o^2 + 2g\Delta d} = \sqrt{0 + 2(10.0\text{ m/s}^2)(321\text{ m})} = \sqrt{6420\text{ m}^2/\text{s}^2}$
$= \mathbf{80.1\ m/s}$

Example 7: The Steamboat Geyser in Yellowstone National Park, Wyoming is capable of shooting its hot water up from the ground with a speed of 48.0 m/s. How high can this geyser shoot?

Solution: Remember, the geyser is shooting **up;** therefore it must have a negative initial velocity.

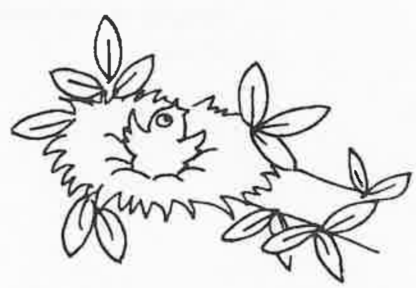
Given: $v_o = -48.0$ m/s
$v_f = 0$ m/s
$g = 10.0$ m/s^2

Unknown: $\Delta d = ?$
Original equation: $v_f^2 = v_o^2 + 2g\Delta d$

Solve: $\Delta d = \dfrac{v_f^2 - v_o^2}{2g} = \dfrac{(0\text{ m/s})^2 - (-48.0\text{ m/s})^2}{2(10.0\text{ m/s}^2)} = \mathbf{-115\ m}$

As you might expect, the final answer has a negative displacement. This means that the total distance the water has traveled is measured up from the ground.

Example 8: A baby blue jay sits in a tall tree awaiting the arrival of its dinner. As the mother lands on the nest, she drops a worm toward the hungry chick's mouth, but the worm misses and falls from the nest to the ground in 1.50 s. How high up is the nest?



Given: $v_o = 0$ m/s
$g = 10.0$ m/s^2
$t = 1.50$ s

Unknown: $\Delta d = ?$
Original equation: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2 = 0 + \left(\frac{1}{2}\right)(10.0\text{ m/s}^2)(1.50\text{ s})^2 = \mathbf{11.3\ m}$

Example 9: A giraffe, who stands 6.00 m tall, bites a branch off a tree to chew on the leaves, and he lets the branch fall to the ground. How long does it take the branch to hit the ground?

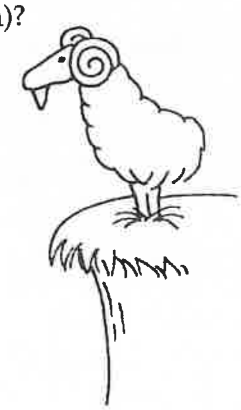
Given: $\Delta d = 6.00$ m
$g = 10.0$ m/s^2
$v_o = 0$ m/s

Unknown: $\Delta t = ?$
Original equation: $\Delta d = v_o\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$

Solve: $\Delta t = \sqrt{\dfrac{2\Delta d}{g}} = \sqrt{\dfrac{2(6.00\text{ m})}{10.0\text{ m/s}^2}} = \sqrt{1.20\text{ s}^2} = \mathbf{1.10\ s}$

Practice Exercises

Exercise 9: Billy, a mountain goat, is rock climbing on his favorite slope one sunny spring morning when a rock comes hurtling toward him from a ledge 40.0 m above. Billy ducks and avoids injury. a) How fast is the rock traveling when it passes Billy? b) How does this speed compare to that of a car traveling down the highway at the speed limit of 25 m/s (equivalent to 55 mi/h)?



Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 10: Reverend Northwick climbs to the church belfry one morning to determine the height of the church. From an outside balcony he drops a book and observes that it takes 2.00 s to strike the ground below. a) How high is the balcony of the church belfry? b) Why would it be difficult to determine the height of the belfry balcony if the Reverend dropped only one page out of the book?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 11: How long is Tina, a ballerina, in the air when she leaps straight up with a speed of 1.8 m/s?

Answer: ______________________

Exercise 12: In order to open the clam it catches, a seagull will drop the clam repeatedly onto a hard surface from high in the air until the shell cracks. If a seagull flies to a height of 25 m, how long will the clam take to fall?

Answer: ____________________

Exercise 13: At Six Flags Great Adventure Amusement Park in New Jersey, a popular ride known as "Free Fall" carries passengers up to a height of 33.5 m and drops them to the ground inside a small cage. How fast are the passengers going at the bottom of this exhilarating journey?

Answer: ____________________

Exercise 14: A unique type of basketball is played on the planet Zarth. During the game, a player flies above the basket and drops the ball in from a height of 10 m. If the ball takes 5.0 s to fall, find the acceleration due to gravity on Zarth.

Answer: ____________________

Additional Exercises

A-1: During an Apollo moon landing, reflecting panels were placed on the moon. This allowed earth-based astronomers to shoot laser beams at the moon's surface to determine its distance. The reflected laser beam was observed 2.52 s after the laser pulse was sent. If the speed of light is 3.00×10^8 m/s, what was the distance between the astronomers and the moon?

A-2: The peregrine falcon is the world's fastest known bird and has been clocked diving downward toward its prey at constant vertical velocity of 97.2 m/s. If the falcon dives straight down from a height of 100. m, how much time does this give a rabbit below to consider his next move as the falcon begins his descent?

A-3: The Kentucky Derby, the first of three horse races for the triple crown, was won on May 7, 2000 by Fusaichi Pegasus with a time of 121.1 s. If the race covers 2011.25 m, what was Fusaichi Pegasus' average speed in a) m/s? b) mi/h?

A-4: For years, the posted highway speed limit was 88.5 km/h (55 mi/h) but now some rural stretches of highway have posted speed limits of 104.6 km/h (65 mi/h). In Maine, the distance from Portland to Bangor is 215 km. How much time can be saved in traveling from Portland to Bangor at this higher speed limit?

A-5: A tortoise and a hare are in a road race to defend the honor of their breeds. The tortoise crawls the entire 1000.-m distance at a speed of 0.2000 m/s while the rabbit runs the first 200.0 m at 2.000 m/s. The rabbit then stops to take a nap for 1.300 h and awakens to finish the last 800.0 m with an average speed of 3.000 m/s. a) Who wins the race and by how much time? b) Draw a graph of distance vs. time for the situation.

A-6: Two physics professors challenge each other to a 100.-m race across the football field. The loser will grade the winner's physics labs for one month. Dr. Nelson runs the race in 10.40 s. Dr. Bray runs the first 25.0 m with an average speed of 10.0 m/s, the next 50.0 m with an average speed of 9.50 m/s, and the last 25.0 m with an average speed of 11.1 m/s. Who gets stuck grading physics labs for the next month?

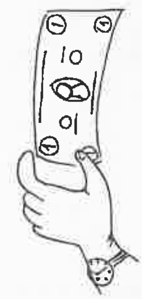
A-7: A caterpillar crawling up a leaf slows from 0.75 cm/s to 0.50 cm/s at a rate of -0.05 cm/s^2. How long does it take the caterpillar to make the change?

A-8: In the Wizard of Oz, Dorothy awakens in Munchkinland where her house has been blown by a tornado. If the house fell from a height of 3000. m, with what speed did it hit the Wicked Witch of the East when it landed?

A-9: The Tambora volcano on the island of Sumbawa, Indonesia has been known to throw ash into the air with a speed of 625 m/s during an eruption. a) How high could this volcanic plume have risen? b) On Venus, where the acceleration due to gravity is slightly less than on Earth, would this volcanic plume rise higher or not as high as it does on Earth?

A-10: Chief Boolie, the jungle dweller, is out hunting for dinner when a coconut falls from a tree and lands on his toe. If the nut fell for 1.4 s, how fast was it traveling when it hit Chief Boolie's toe?

A-11: Here is a bet that you are almost sure to win! Try dropping a dollar bill through a friend's fingers and offer to let her keep it if she can catch it. The bill should be started just at the finger level and your friend shouldn't have any advanced warning when it is going to drop. A dollar bill has a length of 15.5 cm and human reaction time is rarely less than 0.20 s. Do the necessary calculations—why is this almost a sure bet?



A-12: While repairing a defective radio transmitter atop the 410-m-tall World Trade Center, Lyle drops his hammer that falls all the way to the ground below. a) How long will it take for Lyle's hammer to fall? b) With what speed will the hammer hit the pavement? c) How far will the hammer have fallen after 1.50 s when a janitor watches it pass outside an office window?

A-13: On July 31, 1994, Sergey Bubka of the Ukraine broke his own world pole-vaulting record by attaining a height of 6.14 m. a) How long did it take Bubka to return to the ground from the highest part of his vault? b) Describe how this time compares to the time it took him to go from the ground to the highest point.

A-14: A Christmas tree ball will break if dropped on a hardwood floor with a speed of 2.0 m/s or more. Holly is decorating her Christmas tree when her cat, Trickor, taps a ball, causing it to fall 15 cm from a tree branch to the floor. Does the ball break?

A-15: Perhaps sometime in the future, NASA will develop a program to land a human being on Mars. If you were the first Mars explorer and discovered that when you dropped a hammer it took 0.68 s to fall 0.90 m to the ground, what would you calculate for the gravitational acceleration on Mars?

Challenge Exercises for Further Study

B-1: Seth is doing his student driving with the "Give-Me-A-Brake" driving school and is traveling down the interstate with a speed of 9.0 m/s. Mack is driving his "18-wheeler" down the fast lane at 27.0 m/s when he notices Seth 30.0 m ahead of him in the right lane. a) If Mack and Seth maintain their speeds, how far must Mack travel before he catches up to Seth? b) How long will this take?

B-2: At the 2000 summer Olympics in Sydney Australia, the women's 400-m medley swimming relay was won by the United States. The four U.S. women swam the 100.0-m leg of the race with the following average speeds: Barbara Bedford (backstroke) at 1.6289 m/s, Megan Quann (breaststroke) at 1.5085 m/s, Jenny Thompson (fly) at 1.7467 m/s and Dara Torres (freestyle) at 1.8737 m/s. a) How far was the team's final time from the world record time of 4.028 min. set by the Chinese in 1994? b) Did the American women break the world record, or miss it? c) What was the U.S. team's average speed for the 400.0-m race?

B-3: In 1945, the *Enola Gay,* a B-29 bomber, dropped the atomic bomb from a height of 9450 m over Hiroshima, Japan. If the plane carrying the bomb were traveling with a horizontal velocity of 67.0 m/s, how far horizontally would the bomb have traveled between the point of release and the point where it exploded 513 m above the ground? (To avoid being above the bomb when it exploded, the Enola Gay turned sharply away after the bomb's release.)

B-4: Pepe, the clown, is jumping on a trampoline as Babette, the tightrope walker, above him suddenly loses her balance and falls off the tightrope straight toward Pepe. Pepe has just started upward at 15 m/s when Babette begins to fall. Pepe catches her in midair after 1.0 s. a) How far has Babette fallen when she is caught by Pepe? b) What is Babette's velocity at the time of contact? c) What is Pepe's velocity at the time of contact? d) How far above the trampoline was Babette before she fell?

B-5: Mr. DeFronzo has just learned that he won the Presidential Award for Excellence in Science Teaching. He runs to the open window and throws his red marking pen into the air with an initial upward speed of 5.00 m/s. a) If the window is 12.0 m above the ground, what is the velocity of the pen 1.0 s after it is thrown? b) How far has the pen fallen from its starting position after 2.0 s? c) How long does it take the pen to hit the ground?

B-6: On October 24, 1901 Annie Edson Taylor, a school teacher from Michigan, became the first person to successfully ride over Niagara Falls in a wooden barrel. Assume Annie began her journey at Goat Island, 240. m from the falls, where the water current started her down the Niagara River at 8.00 m/s. During her journey, the current reached 15.0 m/s as it carried Annie over Horseshoe Falls, a drop of 51.0 m. How long was Annie's trip from start to finish?

2 Vectors and Projectiles

2-1 Vectors and Scalars

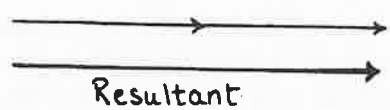
Vocabulary **Vector:** A quantity with magnitude (size) and direction.

Some examples of vectors are displacement, velocity, acceleration, and force.

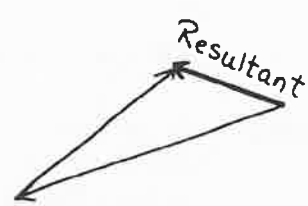
Vocabulary **Scalar:** A quantity with magnitude only.

Some examples of scalars are distance, speed, mass, time, and volume.

Vectors are represented by arrows. They can be added by placing the arrows head to tail. The arrow that extends from the tail of the first vector to the head of the last vector is called the **resultant.** It indicates both the magnitude and direction of the vector sum.

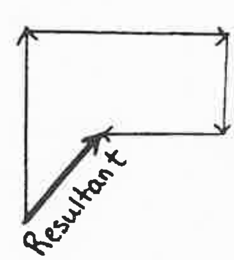


Remember, vectors don't always have to be in a straight line but may be oriented at angles to each other, such as

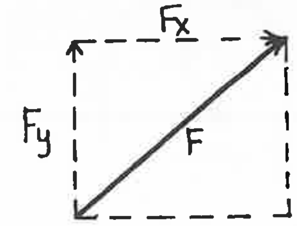


Resultant vectors can be determined by a number of different methods. Here you will solve vector addition exercises both **graphically** and with **vector components.**

Graphical addition of vectors: Using a ruler, draw all vectors to scale and connect them head to tail. The resultant is the vector that connects the tail of the first vector with the head of the last. (Hint: Using graph paper makes this method even easier!)



Vector Components: Because a vector has both magnitude and direction, you can separate it into horizontal (or x) and vertical (or y) components. To do this, draw a rectangle with horizontal and vertical sides and a diagonal equal to the vector. Draw arrow heads on one horizontal and one vertical side to make the original vector the resultant of the horizontal and vertical components.

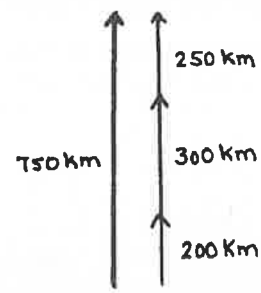


After you have drawn the components, you can find their lengths by using simple trigonometry. If you are not familiar with trigonometry or need a quick refresher, refer to Appendix A.

Solved Examples

Example 1: Every March, the swallows return to San Juan Capistrano, California after their winter in the south. If the swallows fly due north and cover 200 km on the first day, 300 km on the second day, and 250 km on the third day, draw a vector diagram of their trip and find their total displacement for the three-day journey.

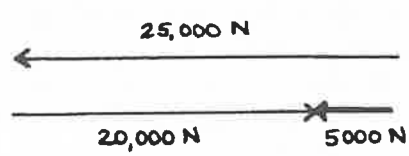
Solution: Because the swallows continue to fly in the same direction throughout the entire trip, these vectors simply add together. This can be shown by placing the displacement vectors head to tail.



200 km + 300 km + 250 km = **750 km north**

Example 2: In the record books, there are men who claim that they have such strong teeth that they can even use them to move cars, trains, and helicopters. Joe Ponder of Love Valley, North Carolina is one such man. Suppose a car pulling forward with a force of 20 000 N was pulled back by a rope that Joe held in his teeth. Joe pulled the car with a force of 25 000 N. Draw a vector diagram of the situation and find the resultant force.

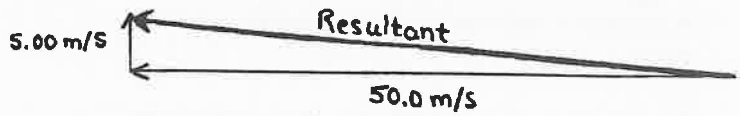
Solution: In this exercise, the vectors are pointing in opposite directions, so the situation would look like this.



25 000 N − 20 000 N = **5000 N in the direction Joe is pulling.** Strong teeth!

Example 3: If St. Louis Cardinals homerun king, Mark McGwire, hit a baseball due west with a speed of 50.0 m/s, and the ball encountered a wind that blew it north at 5.00 m/s, what was the resultant velocity of the baseball?

Solution: Begin by drawing a vector diagram of the situation.



Solve using the Pythagorean theorem:

$$a^2 + b^2 = c^2$$

$$(50.0 \text{ m/s})^2 + (5.00 \text{ m/s})^2 = c^2$$

$$c = \sqrt{2500 \text{ m}^2/\text{s}^2 + 25.0 \text{ m}^2/\text{s}^2} = \mathbf{50.2 \text{ m/s toward the northwest}}$$

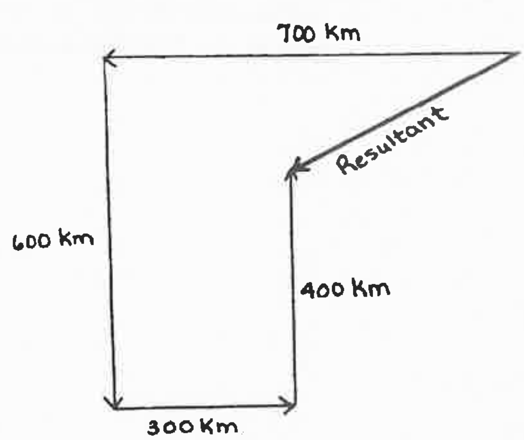
For those of you who understand trigonometry, you can find the exact angle at which the ball travels by saying:

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{5.00 \text{ m/s}}{50.0 \text{ m/s}} = 0.100 \qquad \text{so } \tan^{-1} 0.100 = \mathbf{5.71°}$$

However, don't worry. If you are not familiar with trigonometry, you can simply write the answer as 50.2 m/s to the north of west. For a brief review of trigonometry, see Appendix A.

Example 4: The Maton family begins a vacation trip by driving 700 km west. Then the family drives 600 km south, 300 km east, and 400 north. Where will the Matons end up in relation to their starting point? Solve graphically.

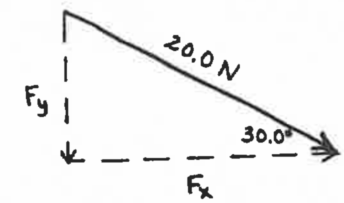
Solution: First, draw the appropriate diagram to scale using a relationship such as 1 cm = 1 km, and you will see a space remaining between where the Matons began their trip and where they ended. Because you are solving this exercise graphically, measure with a ruler the length of the remaining space and convert your measurement back into km. This is the resultant displacement. (Hint: You may find it easier to use graph paper for your drawing so that you can have 1 km equal to a certain number of squares.)



Answer is **450 km,** as measured with a ruler.

Example 5: Ralph is mowing the back yard with a push mower that he pushes downward with a force of 20.0 N at an angle of 30.0° to the horizontal. What are the horizontal and vertical components of the force exerted by Ralph?

Solution: Begin solving by drawing a diagram of the situation, labeling the horizontal and vertical components of the force.



Horizontal component: The hypotenuse in this exercise is the 20.0-N force. The horizontal component is the one going in the *x* direction. This is the side adjacent to the 30.0° angle so you use the equation for the cosine of an angle.

$$\cos\theta = \frac{F_x}{F} \qquad F_x = F\cos\theta = (20.0\text{ N})\cos 30.0° = \mathbf{17.3\ N}$$

Vertical component: Again, the 20.0-N force is the hypotenuse of the triangle. The vertical component is the one going in the y direction. This is the side opposite the 30.0° angle so you use the equation for the sine of an angle.

$$\sin\theta = \frac{F_y}{F} \qquad F_y = F\sin\theta = (20.0\text{ N})\sin 30.0° = \mathbf{10.0\ N}$$

Practice Exercises

Exercise 1: Some Antarctic explorers heading due south toward the pole travel 50. km during the first day. A sudden snow storm slows their progress and they move only 30. km on the second day. With plenty of rest they travel the final 65 km the last day and reach the pole. What was the explorers' displacement?

Answer: ______________________

Exercise 2: Erica and Tory are out fishing on the lake on a hot summer day when they both decide to go for a swim. Erica dives off the front of the boat with a force of 45 N, while Tory dives off the back with a force of 60. N. a) Draw a vector diagram of the situation. b) Find the resultant force on the boat.

Answer: **b.** ____________________

Exercise 3: Young thoroughbreds are sometimes reluctant to enter the starting gate for their first race. Astro Turf is one such horse, and it takes two strong men to get him set for the race. Derek pulls Astro Turf's bridle from the front with a force of 200. N and Dan pushes him from behind with a force of 150. N, while the horse pushes back against the ground with a force of 300. N. a) Draw a vector diagram of the situation. b) What is the resultant force on Astro Turf?

Answer: **b.** ____________________

Exercise 4: Shareen finds that when she drives her motorboat upstream she can travel with a speed of only 8 m/s, while she moves with a speed of 12 m/s when she heads downstream. What is the current of the river on which Shareen is traveling?

Answer: ____________________

Exercise 5: Rochelle is flying to New York for her big Broadway debut. If the plane heads out of Los Angeles with a velocity of 220. m/s in a northeast direction, relative to the ground, and encounters a wind blowing head-on at 45 m/s, what is the resultant velocity of the plane, relative to the ground?

Answer: ____________________

Exercise 6: While Dexter is on a camping trip with his boy scout troop, the scout leader hands each boy a compass and map. The directions on Dexter's map read as follows: "Walk 500.0 m north, 200.0 m east, 300.0 m south, and 400.0 m west." If he follows the map, what is Dexter's displacement? Solve graphically.

Answer: ____________________

Exercise 7: Amit flies due east from San Francisco to Washington, D.C., a displacement of 5600. km. He then flies from Washington to Boston, a displacement of 900. km at an angle of 55.0° east of north. What is Amit's total displacement?

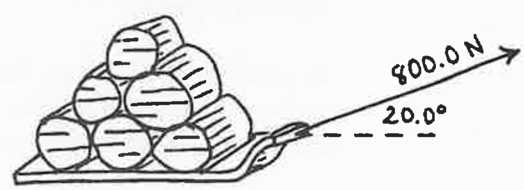
Answer: ____________________

Exercise 8: Marcie shovels snow after a storm by exerting a force of 30.0 N on her shovel at an angle of 60.0° to the vertical. What are the horizontal and vertical components of the force exerted by Marcie?

Answer: ____________________

Answer: ____________________

Exercise 9: Ivan pulls a sled loaded with logs to his cabin in the woods. If Ivan pulls with a force of 800. N in a direction 20.0° above the horizontal, what are the horizontal and vertical components of the force exerted by Ivan?



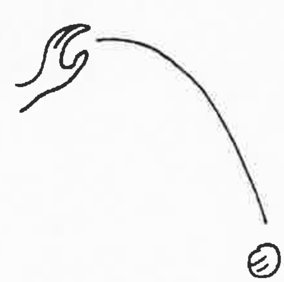
Answer: ____________________

Answer: ____________________

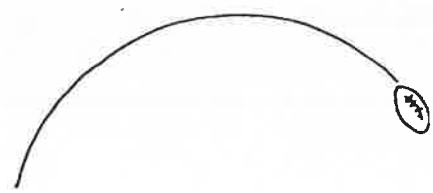
2-2 Projectile Motion

Vocabulary **Projectile:** An object that moves through space acted upon only by the earth's gravity.

A projectile may start at a given height and move toward the ground in an arc. For example, picture the path a rock makes when it is tossed straight out from a cliff.



A projectile may also start at a given level and then move upward and downward again as does a football that has been thrown.



Regardless of its path, a projectile will always follow these rules:

1. Projectiles always maintain a constant horizontal velocity (neglecting air resistance).

2. Projectiles always experience a constant vertical acceleration of 10.0 m/s^2 downward (neglecting air resistance).

3. Horizontal and vertical motion are completely independent of each other. Therefore, the velocity of a projectile can be separated into horizontal and vertical components.

4. For a projectile beginning and ending at the same height, the time it takes to rise to its highest point equals the time it takes to fall from the highest point back to the original position.

5. Objects dropped from a moving vehicle have the same velocity as the moving vehicle.

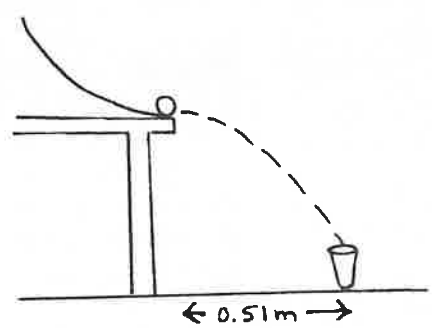
In order to solve projectile exercises, you *must* consider horizontal and vertical motion separately. All of the equations for linear motion in Chapter 1 can be used for projectile motion as well. You don't need to learn any new equations!

To simplify calculations, the term for initial vertical velocity, v_{yo}, will be left out of all equations in which an object is projected horizontally. For example, $\Delta d_y = v_{yo}\Delta t + \left(\frac{1}{2}\right)g\Delta t^2$ will be written as $\Delta d_y = \left(\frac{1}{2}\right)g\Delta t^2$.

Solved Examples

Example 6: In her physics lab, Melanie rolls a 10-g marble down a ramp and off the table with a horizontal velocity of 1.2 m/s. The marble falls in a cup placed 0.51 m from the table's edge. How high is the table?

Solution: The first thing you should notice about projectile exercises is that you do not need to consider the mass of the object projected. Remember, if you ignore air resistance, all bodies fall at exactly the same rate regardless of their mass.



Before you can find the height of the table, you must first determine how long the marble is in the air. The horizontal distance traveled equals the constant horizontal velocity times the travel time.

Given: $\Delta d_x = 0.51\ \text{m}$
$v_x = 1.2\ \text{m/s}$

Unknown: $\Delta t = ?$
Original equation: $v_x = \dfrac{\Delta d_x}{\Delta t}$

Solve: $\Delta t = \dfrac{\Delta d_x}{v_x} = \dfrac{0.51\ \text{m}}{1.2\ \text{m/s}} = 0.43\ \text{s}$

Now that you know the time the marble takes to fall, you can find the vertical distance it traveled.

Given: $g = 10.0\ \text{m/s}^2$
$\Delta t = 0.43\ \text{s}$

Unknown: $\Delta d_y = ?$
Original equation: $\Delta d_y = \left(\dfrac{1}{2}\right)g\Delta t^2$

Solve: $\Delta d_y = \left(\dfrac{1}{2}\right)(10.0\ \text{m/s}^2)(0.43\ \text{s})^2 = \mathbf{0.92\ m}$

Example 7: Bert is standing on a ladder picking apples in his grandfather's orchard. As he pulls each apple off the tree, he tosses it into a basket that sits on the ground 3.0 m below at a horizontal distance of 2.0 m from Bert. How fast must Bert throw the apples (horizontally) in order for them to land in the basket?

Solution: Before you can find the horizontal component of the velocity, you must first find the time that the apple is in the air.

Given: $\Delta d_y = 3.0\ \text{m}$
$g = 10.0\ \text{m/s}^2$

Unknown: $\Delta t = ?$
Original equation: $\Delta d_y = \left(\dfrac{1}{2}\right)g\Delta t^2$

Solve: $t = \sqrt{\dfrac{2\Delta d_y}{g}} = \sqrt{\dfrac{2(3.0\ \text{m})}{10.0\ \text{m/s}^2}} = \mathbf{0.77\ s}$

Now that you know the time, you can use it to find the horizontal component of the velocity.

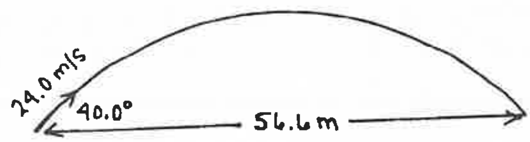
Given: $\Delta d_x = 2.0\ \text{m}$
$\Delta t = 0.77\ \text{s}$

Unknown: $v_x = ?$
Original equation: $\Delta d_x = v_x\,\Delta t$

Solve: $v_x = \dfrac{\Delta d_x}{\Delta t} = \dfrac{2.0\ \text{m}}{0.77\ \text{s}} = \mathbf{2.6\ m/s}$

Example 8: Emanuel Zacchini, the famous human cannonball of the Ringling Bros. and Barnum & Bailey Circus, was fired out of a cannon with a speed of 24.0 m/s at an angle of 40.0° to the horizontal. If he landed in a net 56.6 m away at the same height from which he was fired, how long was Zacchini in the air?

Solution: Because Zacchini was in the air for the same amount of time vertically that he was horizontally, you can find his horizontal time and this will be the answer. First, you need the horizontal velocity component.



$$\cos\theta = \frac{v_x}{v} \qquad v_x = v\cos\theta = (24.0\ \text{m/s})\cos 40.0° = 18.4\ \text{m/s}$$

Now you have the horizontal velocity component and the horizontal displacement, so you can find the time.

Given: $v_x = 18.4\ \text{m/s}$
$\Delta d_x = 56.6\ \text{m}$

Unknown: $\Delta t = ?$
Original equation: $\Delta d_x = v_x \Delta t$

Solve: $\Delta t = \frac{\Delta d_x}{v_x} = \frac{56.6\ \text{m}}{18.4\ \text{m/s}} = \mathbf{3.08\ s}$

Example 9: On May 20, 1999, 37-year old Robbie Knievel, son of famed daredevil Evel Knievel, successfully jumped 69.5 m over a Grand Canyon gorge. Assuming that he started and landed at the same level and was airborne for 3.66 s, what height from his starting point did this daredevil achieve?

Solution: Because 3.66 s is the time for the entire travel through the air, Robbie spent half of this time reaching the height of the jump. The motorcycle took 1.83 s to go up, and another 1.83 s to come down. To find the height the motorcycle achieved, look only at its downward motion as measured from the highest point.

Given: $\Delta t = 1.83\ \text{s}$
$g = 10.0\ \text{m/s}^2$

Unknown: $\Delta d_y = ?$
Original equation: $\Delta d_y = \left(\frac{1}{2}\right) g \Delta t^2$

Solve: $\Delta d_y = \left(\frac{1}{2}\right) g \Delta t^2 = \left(\frac{1}{2}\right)(10.0\ \text{m/s}^2)(1.83\ \text{s})^2 = \mathbf{16.7\ m}$

Practice Exercises

Exercise 10: Billy-Joe stands on the Talahatchee Bridge kicking stones into the water below. a) If Billy-Joe kicks a stone with a horizontal velocity of 3.50 m/s, and it lands in the water a horizontal distance of 5.40 m from where Billy-Joe is standing, what is the height of the bridge? b) If the stone had been kicked harder, how would this affect the time it would take to fall?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 11: The movie "The Gods Must Be Crazy" begins with a pilot dropping a bottle out of an airplane. It is recovered by a surprised native below, who thinks it is a message from the gods. If the plane from which the bottle was dropped was flying at an altitude of 500. m, and the bottle lands 400. m horizontally from the initial dropping point, how fast was the plane flying when the bottle was released?

Answer: ____________________

Exercise 12: Tad drops a cherry pit out the car window 1.0 m above the ground while traveling down the road at 18 m/s. a) How far, horizontally, from the initial dropping point will the pit hit the ground? b) Draw a picture of the situation. c) If the car continues to travel at the same speed, where will the car be in relation to the pit when it lands?

Answer: **a.** ____________________

Answer: **c.** ____________________

Exercise 13: Ferdinand the frog is hopping from lily pad to lily pad in search of a good fly for lunch. If the lily pads are spaced 2.4 m apart, and Ferdinand jumps with a speed of 5.0 m/s, taking 0.60 s to go from lily pad to lily pad, at what angle must Ferdinand make each of his jumps?

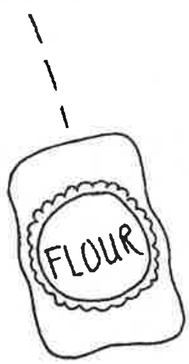
Answer: ____________________

Exercise 14: At her wedding, Jennifer lines up all the single females in a straight line away from her in preparation for the tossing of the bridal bouquet. She stands Kelly at 1.0 m, Kendra at 1.5 m, Mary at 2.0 m, Kristen at 2.5 m, and Lauren at 3.0 m. Jennifer turns around and tosses the bouquet behind her with a speed of 3.9 m/s at an angle of 50.0° to the horizontal, and it is caught at the same height 0.60 s later. a) Who catches the bridal bouquet? b) Who might have caught it if she had thrown it more slowly?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 15: At a meeting of physics teachers in Montana, the teachers were asked to calculate where a flour sack would land if dropped from a moving airplane. The plane would be moving horizontally at a constant speed of 60.0 m/s at an altitude of 300. m. a) If one of the physics teachers neglected air resistance while making his calculation, how far horizontally from the dropping point would he predict the landing? b) Draw a sketch that shows the path the flour sack would take as it falls to the ground (from the perspective of an observer on the ground and off to the side.)



Answer: **a.** ____________________

Exercise 16: Jack be nimble, Jack be quick, Jack jumped over the candlestick with a velocity of 5.0 m/s at an angle of 30.0° to the horizontal. Did Jack burn his feet on the 0.25-m-high candle?

Answer: ____________________

Additional Exercises

A-1: A flock of Canada geese is flying south for the winter. On the first day the geese fly due south a distance of 800. km. On the second day they fly back north 100. km and pause for a couple of days to graze on a sod farm. The last day the geese continue their journey due south, covering a distance of 750. km. a) Draw a vector diagram of the journey and find the total displacement of the geese during this time. b) How does this value differ from the total distance traveled?

A-2: A seal swims toward an inlet with a speed of 5.0 m/s as a current of 1.0 m/s flows in the opposite direction. How long will it take the seal to swim 100. m?

A-3: In Moncton, New Brunswick, each high tide in the Bay of Fundy produces a large surge of water known as a tidal bore. If a riverbed fills with this flowing water that travels north with a speed of 1.0 m/s, what is the resultant velocity of a puffin who tries to swim east across the tidal bore with a speed of 4.0 m/s?

A-4: Lynn is driving home from work and finds that there is road construction being done on her favorite route, so she must take a detour. Lynn travels 5 km north, 6 km east, 3 km south, 4 km west, and 2 km south. a) Draw a vector diagram of the situation. b) What is her displacement? Solve graphically. c) What total distance has Lynn covered?

A-5: Avery sees a UFO out her bedroom window and calls to report it to the police. She says, "The UFO moved 20.0 m east, 10.0 m north, and 30.0 m west before it disappeared." What was the displacement of the UFO while Avery was watching? Solve graphically.

A-6: Eli finds a map for a buried treasure. It tells him to begin at the old oak and walk 21 paces due west, 41 paces at an angle 45° south of west, 69 paces due north, 20 paces due east, and 50 paces at an angle of 53° south of east. How far from the oak tree is the buried treasure? Solve graphically.

A-7: Dwight pulls his sister in her wagon with a force of 65 N at an angle of 50.0° to the vertical. What are the horizontal and vertical components of the force exerted by Dwight?

A-8: Esther dives off the 3-m springboard and initially bounces up with a velocity of 8.0 m/s at an angle of 80.° to the horizontal. What are the horizontal and vertical components of her velocity?

A-9: In many locations, old abandoned stone quarries have become filled with water once excavating has been completed. While standing on a 10.0-m-high quarry wall, Clarence tosses a piece of granite into the water below. If Clarence throws the rock horizontally with a velocity of 3.0 m/s, how far out from the edge of the cliff will it hit the water?

A-10: While skiing, Ellen encounters an unexpected icy bump, which she leaves horizontally at 12.0 m/s. How far out, horizontally, from her starting point will Ellen land if she drops a distance of 7.00 m in the fall?

A-11: The Essex county sheriff is trying to determine the speed of a car that slid off a small bridge on a snowy New England night and landed in a snow pile 4.00 m below the level of the road. The tire tracks in the snow show that the car landed 12.0 m measured horizontally from the bridge. How fast was the car going when it left the road?

A-12: Superman is said to be able to "leap tall buildings in a single bound." How high a building could Superman jump over if he were to leave the ground with a speed of 60.0 m/s at an angle of 75.0° to the horizontal?

A-13: Len is running to school and leaping over puddles as he goes. From the edge of a 1.5-m-long puddle, Len jumps 0.20 m high off the ground with a horizontal velocity component of 3.0 m/s in an attempt to clear it. Determine whether or not Len sits in school all day with wet socks on.

Challenge Exercises for Further Study

B-1: Veronica can swim 3.0 m/s in still water. While trying to swim directly across a river from west to east, Veronica is pulled by a current flowing southward at 2.0 m/s. a) What is the magnitude of Veronica's resultant velocity? b) If Veronica wants to end up exactly across stream from where she began, at what angle to the shore must she swim upstream?

B-2: Solve Practice Exercise A-6 using vector components.

B-3: Mubarak jumps and shoots a field goal from the far end of the court into the basket at the other end, a distance of 27.6 m. The ball is given an initial velocity of 17.1 m/s at an angle of 40.0° to the horizontal from a height of 2.00 m above the ground. What is its velocity as it hits the basket 3.00 m off the ground?

B-4: Drew claims that he can throw a dart at a dartboard from a distance of 2.0 m and hit the 5.0-cm-wide bulls-eye if he throws the dart horizontally with a speed of 15 m/s. He starts the throw at the same height as the top of the bulls-eye. See if Drew is able to hit the bulls-eye by calculating how far his shot falls from the bulls-eye's lower edge.

B-5: Caitlin is playing tennis against a wall. She hits the tennis ball from a height of 0.5 m above the ground with a velocity of 20.0 m/s at an angle of 15.0° to the horizontal toward the wall that is 6.00 m away. a) How far off the ground is the ball when it hits the wall? b) Is the ball still traveling up or is it on its way down when it hits the wall?

B-6: From Chapter 1, Exercise B-6, determine how far from the base of Niagara Falls Annie Taylor landed in her wooden barrel.