4

Momentum

4-1 Impulse and Momentum

Vocabulary

Momentum: A measure of how difficult it is to stop a moving object.

momentum = (mass)(velocity) or
$$p = mv$$

If the momentum of an object is changing, as it is when a force is exerted to start it or stop it, the change in momentum can be found by looking at the change in mass and velocity during the interval.

change in momentum = change in [(mass)(velocity)] or
$$\Delta p = \Delta(mv)$$

For all the exercises in this book, assume that the mass of the object remains constant, and consider only the change in velocity, Δv , which is equal to $v_{\rm f}-v_{\rm o}$. Momentum is a vector quantity. Its direction is in the direction of the object's velocity.

The SI unit for momentum is the kilogram meter/second (kg m/s).

Vocabulary

Impulse: The product of the force exerted on an object and the time interval during which it acts.

impulse = (force)(elapsed time) or
$$J = F\Delta t$$

The SI unit for impulse is the **newton** \cdot **second** (**N** \cdot **s**).

The impulse given to an object is equal to the change in momentum of the object.

$$F\Delta t = m\Delta v$$

The same change in momentum may be the result of a large force exerted for a short time, or a small force exerted for a long time. In other words, impulse is the thing that you *do*, while change in momentum is the thing that you *see*.

The units for impulse and momentum are equivalent. Remember, $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$. Therefore, $1 \text{ N} \cdot \text{s} = 1 \text{ kg} \cdot \text{m/s}$.

Solved Examples

Example 1: Tiger Woods hits a 0.050-kg golf ball, giving it a speed of 75 m/s. What impulse does he impart to the ball?

Solution: Because the impulse equals the change in momentum, you can reword this exercise to read, "What was the ball's change in momentum?" It is understood that the ball was initially at rest, so its initial speed was 0 m/s.

Given:
$$m = 0.050 \text{ kg}$$

 $\Delta v = 75 \text{ m/s}$

Unknown:
$$\Delta p = ?$$

Original equation: $\Delta p = m\Delta v$

Solve:
$$\Delta p = (0.050 \text{ kg})(75 \text{ m/s}) = 3.8 \text{ kg} \cdot \text{m/s}$$

Example 2: Wayne hits a stationary 0.12-kg hockey puck with a force that lasts for 1.0×10^{-2} s and makes the puck shoot across the ice with a speed of 20.0 m/s, scoring a goal for the team. With what force did Wayne hit the puck?

Given:
$$m = 0.12 \text{ kg}$$

 $\Delta v = 20.0 \text{ m/s}$
 $\Delta t = 1.0 \times 10^{-2} \text{ s}$

Unknown:
$$F = ?$$
Original equation: $F\Delta t = m\Delta v$

Solve:
$$F = \frac{m\Delta v}{\Delta t} = \frac{(0.12 \text{ kg})(20.0 \text{ m/s})}{1.0 \times 10^{-2} \text{ s}} = 240 \text{ kg} \cdot \text{m/s}^2 = 240 \text{ N}$$

A tennis ball traveling at 10.0 m/s is returned by Venus Williams. It leaves her racket with a speed of 36.0 m/s in the opposite direction from which it came.

a) What is the change in momentum of the tennis ball? b) If the 0.060-kg ball is in contact with the racket for 0.020 s, with what average force has Venus hit the ball?

Solution: In this exercise, the tennis ball is coming toward Venus with a speed of 10.0 m/s in one direction, but she hits it back with a speed of 36.0 m/s in the opposite direction. Therefore, you must think about velocity vectors and call one direction positive and the opposite direction negative.

a. Given:
$$v_0 = -10.0 \text{ m/s}$$

 $v_f = 36.0 \text{ m/s}$
 $m = 0.060 \text{ kg}$

Unknown:
$$\Delta p = ?$$

Original equation: $\Delta p = m\Delta v = m(v_{\rm f} - v_{\rm o})$

Solve:
$$\Delta p = m(v_f - v_o) = (0.060 \text{ kg})[36.0 \text{ m/s} - (-10.0 \text{ m/s})] = 2.8 \text{ kg} \cdot \text{m/s}$$

b. Given:
$$m = 0.060 \text{ kg}$$
 Unknown: $F = ?$

$$\Delta v = 46.0 \text{ m/s}$$
 Original equation: $F\Delta t = m\Delta v$

$$\Delta t = 0.020 \text{ s}$$

Solve:
$$F = \frac{m\Delta v}{\Delta t} = \frac{(0.060 \text{ kg})(46.0 \text{ m/s})}{(0.020 \text{ s})} = 140 \text{ N}$$

Example 4: To demonstrate his new high-speed camera, Flash performs an experiment in the physics lab in which he shoots a pellet gun at a pumpkin to record the moment of impact on film. The 1.0-g pellet travels at 100. m/s until it embeds itself 2.0 cm into the pumpkin. What average force does the pumpkin exert to stop the pellet?

Solution: First, convert g to kg and cm to m.

$$1.0 \text{ g} = 0.0010 \text{ kg}$$
 $2.0 \text{ cm} = 0.020 \text{ m}$

Before you can solve for the force in the exercise, you must first know how long the force is being exerted. Remember, in order to find the time, you must use the average velocity, v_{av} .

$$v_{\rm av} = \frac{v_{\rm f} + v_{\rm o}}{2} = \frac{0 \text{ m/s} + 100. \text{ m/s}}{2} = 50.0 \text{ m/s}$$

Given:
$$v = 50.0 \text{ m/s}$$

 $\Delta d = 0.020 \text{ m}$

Unknown:
$$\Delta t = ?$$
 Original equation: $\Delta d = v\Delta t$

Solve:
$$\Delta t = \frac{\Delta d}{v} = \frac{0.020 \text{ m}}{50.0 \text{ m/s}} = 0.00040 \text{ s}$$

Now we can solve for the force the pumpkin exerts to stop the pellet.

Given:
$$m = 0.0010 \text{ kg}$$

 $\Delta v = 100. \text{ m/s}$
 $\Delta t = 0.0040 \text{ s}$

Unknown:
$$F = ?$$
 Original equation: $F\Delta t = m\Delta v$

Solve:
$$F = \frac{m\Delta v}{\Delta t} = \frac{(0.0010 \text{ kg})(100. \text{ m/s})}{(0.00040 \text{ s})} = 250 \text{ N}$$

Practice Exercises

On April 15, 1912, the luxury cruiseliner Titanic sank after running into an Exercise 1: iceberg. a) What momentum would the 4.23×10^8 -kg ship have imparted to the iceberg if it had hit the iceberg head-on with a speed of 11.6 m/s? (Actually, the impact was a glancing blow.) b) If the captain of the ship had seen the iceberg a kilometer ahead and had tried to slow down, why would this have been a futile effort?

Answer: a. _____

Answer: **b.** _____

Exercise 2:	Auto companies frequently test the safety of automobiles by putting them through crash tests to observe the integrity of the passenger compartment. If a 1000kg car is sent toward a cement wall with a speed of 14 m/s and the impact brings it to a stop in 8.00×10^{-2} s, with what average force is it brought to rest?
	Answer:
Exercise 3:	Rhonda, who has a mass of 60.0 kg, is riding at 25.0 m/s in her sports car when she must suddenly slam on the brakes to avoid hitting a dog crossing the road. She is wearing her seatbelt, which brings her body to a stop in 0.400 s. a) What average force did the seatbelt exert on her? b) If she had not been wearing her seatbelt, and the windshield had stopped her head in 1.0×10^{-3} s, what average force would the windshield have exerted on her? c) How many times greater is the stopping force of the windshield than the seatbelt?
	Answer: a.
	Answer: b.
	Answer: c.
Exercise 4:	If 270 million people in the United States jumped up in the air simultaneously pushing off Earth with an average force of 800. N each for a time of 0.10 s, what would happen to the 5.98×10^{24} kg Earth? Show a calculation that justifies your answer.
	Answer:

In Sharkey's Billiard Academy, Maurice is waiting to make his last shot. He Exercise 5: notices that the cue ball is lined up for a perfect head-on collision, as shown. Each of the balls has a mass of 0.0800 kg and the cue ball comes to a complete stop upon making contact with the 8 ball. Suppose Maurice hits the cue ball by exerting a force of 180. N for 5.00×10^{-3} s, and it knocks head-on into the 8 ball. Calculate the resulting velocity of the 8 ball.



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Answer:	

Exercise 6: During an autumn storm, a 0.012-kg hail stone traveling at 20.0 m/s made a 0.20-cm-deep dent in the hood of Darnell's new car. What average force did the car exert to stop the damaging hail stone?

A	
Answer:	

4-2 **Conservation of Momentum**

According to the law of conservation of momentum, the total momentum in a system remains the same if no external forces act on the system. Consider the two types of collisions that can occur.

Vocabulary

Elastic collision: A collision in which objects collide and bounce apart with no energy loss.

In an elastic collision, because momentum is conserved, the mv before a collision for each of the two objects must equal the mv after the collision for each of the two objects. This is written as

$$m_1 v_{1o} + m_2 v_{2o} = m_1 v_{1f} + m_2 v_{2f}$$

The subscripts 1 and 2 refer to objects 1 and 2, respectively.

Vocabulary

Inelastic collision: A collision in which objects collide and some mechanical energy is transformed into heat energy.

A common kind of inelastic collision is one in which the colliding objects stick together, or start out stuck together and then separate. However, in an inelastic collision the objects need not remain stuck together but may instead deform in some way.

Because momentum is also conserved in an inelastic collision, the *mv* before the collision for each of the two objects must equal the *mv* after the collision for each of the two objects. When objects are stuck together after the collision (assuming mass does not change), this equation becomes

$$m_1 v_{10} + m_2 v_{20} = (m_1 + m_2) v_{\rm f}$$

where v_f is the combined final velocity of the two objects.

Solved Examples

Example 5:

Tubby and his twin brother Chubby have a combined mass of 200.0 kg and are zooming along in a 100.0-kg amusement park bumper car at 10.0 m/s. They bump Melinda's car, which is sitting still. Melinda has a mass of 25.0 kg. After the elastic collision, the twins continue ahead with a speed of 4.12 m/s. How fast is Melinda's car bumped across the floor?

Solution: Notice that you must add the mass of the bumper car to the mass of the riders.

Sometimes the curiosity factor at the scene of a car accident is so great that it Example 6: actually produces secondary accidents as a result, while people watch to see what is going on. If an 800.-kg sports car slows to 13.0 m/s to check out an accident scene and the 1200.-kg pick-up truck behind him continues traveling at 25.0 m/s, with what velocity will the two move if they lock bumpers after a rear-end collision?

> Solution: Since the two vehicles lock bumpers, both objects have the same final velocity.

Solve:
$$v_{\rm f} = \frac{m_1 v_{10} + m_2 v_{20}}{(m_1 + m_2)} = \frac{(800. \text{ kg})(13.0 \text{ m/s}) + (1200. \text{ kg})(25.0 \text{ m/s})}{(800. \text{ kg} + 1200. \text{ kg})}$$
$$= \frac{10 \, 400 \, \text{kg} \cdot \text{m/s} + 30 \, 000 \, \text{kg} \cdot \text{m/s}}{2000. \text{ kg}} = 20.2 \, \text{m/s forward}$$

Charlotte, a 65.0-kg skin diver, shoots a 2.0-kg spear with a speed of 15 m/s at Example 7: a fish who darts quickly away without getting hit. How fast does Charlotte move backwards when the spear is shot?

Solution: To start, Charlotte and the spear are together and both are at rest.

Remember, the minus sign here is indicating direction. Therefore, Charlotte would travel with a speed of 0.46 m/s in a direction opposite to that of the spear.



Practice Exercises

Exercise 7:	Jamal is at the state fair playing some of the games. At one booth he throws a 0.50-kg ball forward with a velocity of 21.0 m/s in order to hit a 0.20-kg bottle sitting on a shelf, and when he makes contact the bottle goes flying forward at 30.0 m/s. a) What is the velocity of the ball after it hits the bottle? b) If the bottle were more massive, how would this affect the final velocity of the ball?
	Answer: a.
	Answer: b.
Exercise 8:	Jeanne rolls a 7.0-kg bowling ball down the alley for the league championship. One pin is still standing, and Jeanne hits it head-on with a velocity of 9.0 m/s. The 2.0-kg pin acquires a forward velocity of 14.0 m/s. What is the new velocity of the bowling ball?
	Answer:
Evercise 9.	Running at 2.0 m/s, Bruce, the 45.0-kg quarterback, collides with Biff, the

Exercise 9: Running at 2.0 m/s, Bruce, the 45.0-kg quarterback, collides with Biff, the 90.0-kg tackle, who is traveling at 7.0 m/s in the other direction. Upon collision, Biff continues to travel forward at 1.0 m/s. How fast is Bruce knocked backwards?

Answer: ____

Exercise 10:	Anthony and Sissy are participating in the "Roll-a-Rama" rollerskating dance championship. While 75.0-kg Anthony rollerskates backwards at 3.0 m/s, 60.0-kg Sissy jumps into his arms with a velocity of 5.0 m/s in the same direction. a) How fast does the pair roll backwards together? b) If Anthony is skating toward Sissy when she jumps, would their combined final velocity be
	skating toward Sissy when she jumps, would their combined final velocity be larger or smaller than your answer to part a? Why?

Answer:	a		
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To test the strength of a retainment wall designed to protect a nuclear reactor, Exercise 11: a rocket-propelled F-4 Phantom jet aircraft was crashed head-on into a concrete barrier at high speed in Sandia, New Mexico on April 19, 1988. The F-4 phantom had a mass of 19100 kg, while the retainment wall's mass was 469000 kg. The wall sat on a cushion of air that allowed it to move during impact. If the wall and F-4 moved together at 8.41 m/s during the collision, what was the initial speed of the F-4 Phantom?

Answer:

Exercise 12:	Valentina, the Russian Cosmonaut, goes outside her ship for a spacewalk, but when she is floating 15 m from the ship, her tether catches on a sharp piece of metal and is severed. Valentina tosses her 2.0-kg camera away from the spaceship with a speed of 12 m/s. a) How fast will Valentina, whose mass is now 68 kg, travel toward the spaceship? b) Assuming the spaceship remains at rest with respect to Valentina, how long will it take her to reach the ship?
	Answer: a
	Answer: b
Exercise 13:	A 620kg moose stands in the middle of the railroad tracks, frozen by the lights of an oncoming 10 000kg locomotive that is traveling at 10.0 m/s. The engineer sees the moose but is unable to stop the train in time and the moose rides down the track sitting on the cowcatcher. What is the new combined velocity of the locomotive and the moose?
	E MIN
	Answer:
Exercise 14:	Lee is rolling along on her 4.0-kg skateboard with a constant speed of 3.0 m/s when she jumps off the back and continues forward with a velocity of 2.0 m/s relative to the ground. This causes the skateboard to go flying forward with a speed of 15.5 m/s relative to the ground. What is Lee's mass?
	Answer:

Additional Exercises

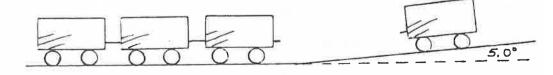
- Bernie, whose mass is 70.0 kg, leaves a ski jump with a velocity of 21.0 m/s. A-1: What is Bernie's momentum as he leaves the ski jump?
- A-2: Ethel is sitting on a park bench feeding the pigeons when a child's ball rolls toward her across the grass. Ethel returns the ball to the child by hitting it with her 2.0-kg pocketbook with a speed of 20 m/s. If the impact lasts for 0.4 s, with what force does Ethel hit the ball?
- A-3: When Reggie stepped up to the plate and hit a 0.150-kg fast ball traveling at 36.0 m/s, the impact caused the ball to leave his bat with a velocity of 45.0 m/s in the opposite direction. If the impact lasted for 0.002 s, what force did Reggie exert on the baseball?
- The U.S. Army's parachuting team, the Golden Knights, are on a routine A-4: jumping mission over a deserted beach. On a jump, a 65-kg Knight lands on the beach with a speed of 4.0 m/s, making a 0.20-m deep indentation in the sand. With what average force did the parachuter hit the sand?
- A-5: The late news reports the story of a shooting in the city. Investigators think that they have recovered the weapon and they run ballistics tests on the pistol at the firing range. If a 0.050-kg bullet were fired from the handgun with a speed of 400 m/s and it traveled 0.080 m into the target before coming to rest, what force did the bullet exert on the target?
- About 50 000 years ago, in an area located outside Flagstaff, Arizona, a giant A-6: 4.5×10^7 -kg meteor fell and struck the earth, leaving a 180-m-deep hole now known as Barringer crater. If the meteor was traveling at 20 000 m/s upon impact, with what average force did the meteor hit the earth?
- A-7: Astronaut Pam Melroy, history's third woman space shuttle pilot, flew the space shuttle Discovery to the International Space Station to complete construction in October of 2000. To undock from the space station Pilot Melroy released hooks holding the two spacecraft together and the 68 000-kg shuttle pushed away from the space station with the aid of four large springs. a) If the 73 000-kg space station moved back at a speed of 0.50 m/s, how fast and in what direction did the space shuttle move? b) What was the relative speed of the two spacecraft as they separated?
- A-8: Tyrrell throws his 0.20-kg football in the living room and knocks over his mother's 0.80-kg antique vase. After the collision, the football bounces straight back with a speed of 3.9 m/s, while the vase is moving at 2.6 m/s in the opposite direction. a) How fast did Tyrrell throw the football? b) If the football continued to travel at 3.9 m/s in the same direction it was thrown, would the vase have to be more or less massive than 0.80 kg?
- A-9: A 300.-kg motorboat is turned off as it approaches a dock and it coasts in toward the dock at 0.50 m/s. Isaac, whose mass is 62.0 kg, jumps off the front

of the boat with a speed of 3.0 m/s relative to the boat. What is the velocity of the boat after Isaac jumps?

- A-10: Miguel, the 72.0-kg bullfighter, runs toward an angry bull at a speed of 4.00 m/s. The 550.-kg bull charges toward Miguel at 12.0 m/s and Miguel must jump on the bull's back at the last minute to avoid being run over. What is the new velocity of Miguel and the bull as they move across the arena?
- A-11: A space shuttle astronaut is sent to repair a defective relay in a 600.00-kg satellite that is traveling in space at 17 000.0 m/s. Suppose the astronaut and his Manned Maneuvering Unit (MMU) have a mass of 400.00 kg and travel at 17 010.0 m/s toward the satellite. What is the combined velocity when the astronaut grabs hold of the satellite?
- A-12: The U.S.S. Constitution, the oldest fully commissioned war ship in the world, is docked in Boston, Massachusetts. Also known as "Old Ironsides" for her seemingly impenetrable hull, the frigate houses 56 pieces of heavy artillery. Mounted on bearings that allow them to recoil at a speed of 1.30 m/s are 20 carronades, each with a mass of 1000. kg. If a carronade fires a 14.5-kg cannonball straight ahead, with what muzzle velocity does the cannonball leave the cannon?

Challenge Exercises for Further Study

- B-1: On a hot sumer afternoon, Keith and Nate are out fishing in their rowboat when they decide to jump into the water and go for a swim. Keith, whose mass is 65.0 kg, jumps straight off the front of the boat with a speed of 2.00 m/s relative to the boat, while Nate propels his 68.0-kg body simultaneously off the back of the boat at 4.00 m/s relative to the boat. If the 100.-kg boat is initially traveling forward at 3.00 m/s, what is its velocity after both boys jump?
- B-2: Lilly, whose mass is 45.0 kg, is ice skating with a constant speed of 7.00 m/s when she hits a rough patch of ice with a coefficient of friction of 0.0800. How long will it take before Lilly coasts to a stop?
- B-3: In a train yard, train cars are rolled down a long hill in order to link them up with other cars as shown. A car of mass 4000. kg starts to roll from rest at the top of a hill 5.0 m high, and inclined at an angle of 5.0° to the horizontal. The coefficient of rolling friction between the train and the track is 0.050. What velocity would the car have if it linked up with 3 identical cars sitting on flat ground at the bottom of the track? (Hint: The equation for rolling friction is just like the one for sliding friction.)



5

Energy and Machines

5-1 Work and Power

Vocabulary

Work: The product of the component of the force exerted on an object in the direction of displacement and the magnitude of the displacement.

or
$$W = F\Delta d$$

The SI unit for work is the **joule** (J), which equals one newton \cdot meter (N \cdot m).

For maximum work to be done, the object *must* move in the direction of the force. If the object is moving at an angle to the force, determine the component of the force in the direction of motion. Remember, if the object does not move, or moves perpendicular to the direction of the force, no work has been done.

Vocabulary

Power: The rate at which work is done.

$$\mathbf{power} = \frac{\mathbf{work}}{\mathbf{elapsed\ time}} \qquad \mathbf{or} \qquad P = \frac{W}{\Delta t}$$

The SI unit for power is the watt (W), which equals one joule per second (J/s). One person is more powerful than another if he or she can do more work in a given amount of time, or can do the same amount of work in less time.

Solved Examples

Example 1:

Bud, a very large man of mass 130 kg, stands on a pogo stick. How much work is done as Bud compresses the spring of the pogo stick 0.50 m?

Solution: First, find Bud's weight, which is the force with which he compresses the pogo stick spring.

Given:
$$m = 130 \text{ kg}$$

 $g = 10.0 \text{ m/s}^2$

$$Unknown: w = ?$$

Original equation:
$$w = mg$$

Solve:
$$w = mg = (130 \text{ kg})(10.0 \text{ m/s}^2) = 1300 \text{ N}$$

Now use this weight to solve for the work done to compress the spring.

Given: F = 1300 N Unknown: W = ?

 $\Delta d = 0.050 \text{ m}$ Original equation: $W = F\Delta d$

Solve: $W = F\Delta d = (1300 \text{ N})(0.050 \text{ m}) = 65 \text{ J}$

Don't get confused here by the two W's you see in this example. The w in w = mg means weight while the W in $W = F\Delta d$ means work. There are many ways to tell them apart, the most important of which is to understand how they are used in the context of the exercise. Also, the units used for each are quite different: weight is measured in newtons, and work is measured in joules. Last of all, weight is a vector and work is a scalar.

Example 2: After finishing her physics homework, Sherita pulls her 50.0-kg body out of the living room chair and climbs up the 5.0-m-high flight of stairs to her bedroom. How much work does Sherita do in ascending the stairs?

Solution: First find Sherita's weight. Her muscles exert a force to carry her weight up the stairs.

Given: m = 50.0 kg Unknown: w = ?

 $g = 10.0 \text{ m/s}^2$ Original equation: w = mg

Solve: $w = mg = (50.0 \text{kg})(10.0 \text{ m/s}^2) = 500. \text{ N}$

Now use Sherita's weight (or force) to determine the amount of work done. It is important to note that when you are solving for the work done, you need know only the displacement of the body moved. The number of stairs climbed or their steepness is irrelevant. All that is important is the *change* in position.

Solve: $W = F\Delta d = (500. \text{ N})(5.0 \text{ m}) = 2500 \text{ J}$

Example 3: In the previous example, Sherita slowly ascends the stairs, taking 10.0 s to go from bottom to top. The next evening, in a rush to catch her favorite TV show, she runs up the stairs in 3.0 s. a) On which night does Sherita do more work? b) On which night does Sherita generate more power?

- a) Sherita does the same amount of work on both nights because the force she exerts and her displacement are the same each time.
- b) Sherita's power output varies because the time taken to do the same amount of work varies.

First night:

Given: W = 2500 J Unknown: P = ? $\Delta t = 10.0 \text{ s}$ Original equation: $P = \frac{W}{\Delta t}$

Solve:
$$P = \frac{W}{\Delta t} = \frac{2500 \text{ J}}{10.0 \text{ s}} = 250 \text{ W}$$

Second night:

Given:
$$W = 2500 \text{ J}$$

 $\Delta t = 3.0 \text{ s}$

Unknown:
$$P = ?$$

Original equation: $P = \frac{W}{\Delta t}$

Solve:
$$P = \frac{W}{\Delta t} = \frac{2500 \text{ J}}{3.0 \text{ s}} = 830 \text{ W}$$

Sherita generates more power on the second night.

Practice Exercises

Exercise 1: On his way off to college, Russell drags his suitcase 15.0 m from the door of his house to the car at a constant speed with a horizontal force of 95.0 N. a) How much work does Russell do to overcome the force of friction? b) If the floor has just been waxed, does he have to do more work or less work to move the suitcase? Explain.

Answer: a. _

Answer: b.

Katie, a 30.0-kg child, climbs a tree to rescue her cat who is afraid to jump Exercise 2: 8.0 m to the ground. How much work does Katie do in order to reach the cat?

Answer: _

Exercise 3: Marissa does 3.2 J of work to lower the window shade in her bedroom a distance of 0.8 m. How much force must Marrisa exert on the window shade?

Answer:	
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Exercise 4: Atlas and Hercules, two carnival sideshow strong men, each lift 200.-kg barbells 2.00 m off the ground. Atlas lifts his barbells in 1.00 s and Hercules lifts his in 3.00 s. a) Which strong man does more work? b) Calculate which

man is more powerful.

Answer: a.	
Answer: b.	

5-2 Energy

Potential and Kinetic Energy

Vocabulary Energy: The ability to do work.

There are many different types of energy. This chapter will focus on only mechanical energy, or the energy related to position (**potential energy**) and motion (**kinetic energy**).

Vocabulary Potential Energy: Energy of position, or stored energy.

An object gains gravitational potential energy when it is lifted from one level to a higher level. Therefore, we generally refer to the *change* in potential energy or ΔPE , which is proportional to the change in height, Δh .

 Δ gravitational potential energy = (mass)(acceleration due to gravity)(Δ height)

or
$$\Delta PE = mg\Delta h$$

It is important to remember that gravitational potential energy relies *only* upon the vertical change in height, Δh , and not upon the path taken.

In addition to gravitational potential energy, there are other forms of stored energy. For example, when a bow is pulled back and before it is released, the energy in the bow is equal to the work done to deform it. This stored or potential energy is written as $\Delta PE = F\Delta d$. Springs possess elastic potential energy when they are displaced from the equilibrium position. The equation for elastic potential energy will not be used in this chapter.

Vocabulary Kinetic Energy: Energy of motion.

The kinetic energy of an object varies with the square of the speed.

kinetic energy =
$$\left(\frac{1}{2}\right)$$
 (mass)(speed)² or KE = $\left(\frac{1}{2}\right)$ mv^2

The SI unit for energy is the **joule**. Notice that this is the same unit used for work. When work is done on an object, energy is transformed from one form to another. The sum of the changes in potential, kinetic, and heat energy is equal to the work done on the object. Mechanical energy is transformed into heat energy when work is done to overcome friction.

Conservation of Energy

According to the **law of conservation of energy**, energy cannot be created or destroyed. The total amount of mechanical energy in a system remains constant if no work is done by any force other than gravity.

In an isolated system where there are no mechanical energy losses due to friction

$$\Delta KE = \Delta PE$$

In other words, all the kinetic and potential energy before an interaction equals all the kinetic and potential energy after the interaction.

$$\mathrm{KE_o} + \mathrm{PE_o} = \mathrm{KE_f} + \mathrm{PE_f}$$
 or $\left(\frac{1}{2}\right) m{v_o}^2 + mgh_o = \left(\frac{1}{2}\right) m{v_f}^2 + mgh_f$

As a reminder, the terms with the subscript $_{\rm o}$ are the initial conditions, while those with the subscript $_{\rm f}$ are final conditions.

Solved Examples

Example 4: Legend has it that Isaac Newton "discovered" gravity when an apple fell from a tree and hit him on the head. If a 0.20-kg apple fell 7.0 m before hitting Newton, what was its change in PE during the fall?

Solution: For a given object, the change in PE depends only upon the change in position. The apple does not need to fall all the way to the ground to experience an energy change.

Given:
$$m = 0.20 \text{ kg}$$
 Unknown: $\Delta PE = ?$ $g = 10.0 \text{ m/s}^2$ Original equation: $\Delta PE = mg\Delta h$ $\Delta h = 7.0 \text{ m}$

Solve: $\Delta PE = mg\Delta h = (0.20 \text{ kg})(10.0 \text{ m/s}^2)(7.0 \text{ m}) = 14 \text{ J}$

Example 5: A greyhound at a race track can run at a speed of 16.0 m/s. What is the KE of a 20.0-kg greyhound as it crosses the finish line?

Given:
$$m = 20.0 \text{ kg}$$
 Unknown: $KE = ?$ Original equation: $KE = \left(\frac{1}{2}\right)mv^2$

Solve: $KE = \left(\frac{1}{2}\right)mv^2 = \left(\frac{1}{2}\right)(20.0 \text{ kg})(16.0 \text{ m/s})^2 = 2560 \text{ J}$

Example 6: In a wild shot, Bo flings a pool ball of mass m off a 0.68-m-high pool table, and the ball hits the floor with a speed of 6.0 m/s. How fast was the ball moving when it left the pool table? (Use the law of conservation of energy.)

Solve:
$$KE_o + PE_o = KE_f + PE_f$$
 or $\left(\frac{1}{2}\right)mv_o^2 + mgh_o = \left(\frac{1}{2}\right)mv_f^2 + mgh_f$

Notice that mass is contained in each of these equations. Therefore, it cancels out and does not need to be included in the calculation.

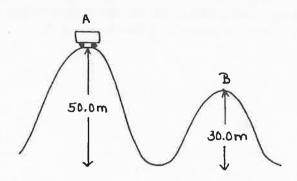
$$v_{o} = \sqrt{\frac{\left(\frac{1}{2}\right)mv_{f}^{2} + mgh_{f} - mgh_{o}}{\left(\frac{1}{2}\right)m}} = \sqrt{\frac{\left(\frac{1}{2}\right)v_{f}^{2} + gh_{f} - gh_{o}}{\frac{1}{2}}}$$

$$= \sqrt{\frac{\left(\frac{1}{2}\right)(6.0 \text{ m/s})^{2} + (10.0 \text{ m/s}^{2})(0 \text{ m}) - (10.0 \text{ m/s}^{2})(0.68 \text{ m})}{\frac{1}{2}}}$$

$$= \sqrt{\frac{18 \text{ m}^{2}/\text{s}^{2} - 6.8 \text{ m}^{2}/\text{s}^{2}}{\frac{1}{2}}} = 4.7 \text{ m/s}$$

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Example 7: Frank, a San Francisco hot dog vender, has fallen asleep on the job. When an earthquake strikes, his 300-kg hot-dog cart rolls down Nob Hill and reaches point A at a speed of 8.00 m/s. How fast is the hot-dog cart going at point B when Frank finally wakes up and starts to run after it?



Solution: Because mass is contained in each of these equations, it cancels out and does not need to be included in the calculation. Also, the inclination of the hill makes no difference. All that matters is the change in height.

Given:
$$v_o = 8.00 \text{ m/s}$$

 $g = 10.0 \text{ m/s}^2$
 $h_o = 50.0 \text{ m}$
 $h_f = 30.0 \text{ m}$

Unknown:
$$v_f = ?$$

Original equation: $\Delta KE = \Delta PE$

Solve:
$$KE_o + PE_o = KE_f + PE_f$$
 or $\left(\frac{1}{2}\right)m{v_o}^2 + mgh_o = \left(\frac{1}{2}\right)m{v_f}^2 + mgh_f$

$$v_{\rm f} = \sqrt{\frac{\left(\frac{1}{2}\right)mv_{\rm o}^2 + mgh_{\rm o} - mgh_{\rm f}}{\left(\frac{1}{2}\right)m}} = \sqrt{\frac{\left(\frac{1}{2}\right)v_{\rm o}^2 + gh_{\rm o} - gh_{\rm f}}{\frac{1}{2}}}$$

$$= \sqrt{\frac{\left(\frac{1}{2}\right)(8.00 \text{ m/s})^2 + (10.0 \text{ m/s}^2)(50.0 \text{ m}) - (10.0 \text{ m/s}^2)(30.0 \text{ m})}{\frac{1}{2}}}$$

$$= \sqrt{\frac{32.0 \text{ m}^2/\text{s}^2 + 500. \text{ m}^2/\text{s}^2 - 300. \text{ m}^2/\text{s}^2}{\frac{1}{2}}}$$

$$= \sqrt{464 \text{ m}^2/\text{s}^2} = 21.5 \text{ m/s}$$

Practice Exercises

Exercise 5:	It is said that Galileo dropped objects off the Leaning Tower of Pisa to determine whether heavy or light objects fall faster. If Galileo had dropped a 5.0-kg cannon ball to the ground from a height of 12 m, what would have been the change in PE of the cannon ball?
	Answer:
Exercise 6:	The 2000 Belmont Stakes winner, Commendable, ran the horse race at an average speed of 15.98 m/s. If Commendable and jockey Pat Day had a combined mass of 550.0 kg, what was their KE as they crossed the finish line?
	Answer:
Exercise 7:	Brittany is changing the tire of her car on a steep hill 20.0 m high. She trips and drops the 10.0-kg spare tire, which rolls down the hill with an initial speed of 2.00 m/s. What is the speed of the tire at the top of the next hill, which is 5.00 m high? (Ignore the effects of rotation KE and friction.)

Exercise 8:	A Mexican jumping bean jumps with the aid of a small worm that lives inside the bean. a) If a bean of mass 2.0 g jumps 1.0 cm from your hand into the air, how much potential energy has it gained in reaching its highest point. b) What is its speed as the bean lands back in the palm of your hand?
	Answer: b.
Exercise 9:	A 500kg pig is standing at the top of a muddy hill on a rainy day. The hill is 100.0 m long with a vertical drop of 30.0 m. The pig slips and begins to slide down the hill. What is the pig's speed at the bottom of the hill? Use the law of conservation of energy.
	Answer:
Exercise 10:	While on the moon, the Apollo astronauts enjoyed the effects of a gravity much smaller than that on Earth. If Neil Armstrong jumped up on the moon with an initial speed of 1.51 m/s to a height of 0.700 m, what amount of gravitational acceleration did he experience?

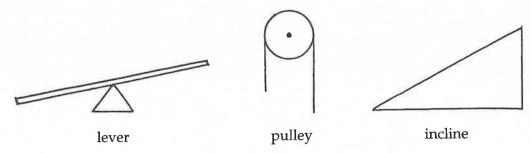
Answer:

5-3 Machines and Efficiency

Vocabulary

Machine: A device that helps do work by changing the magnitude or direction of the applied force.

Three common machines are the lever, pulley, and incline.



In an ideal situation, where frictional forces are negligible, work input equals work output.

$$F_{\rm in}\Delta d_{\rm in} = F_{\rm out}\Delta d_{\rm out}$$

However, situations are never ideal. The **actual mechanical advantage**, or **AMA**, of the machine is a ratio of the magnitude of the force out (resistance) to the magnitude of the force in (effort).

actual mechanical advantage =
$$\frac{\text{force out (resistance)}}{\text{force in (effort)}}$$
 or $AMA = \frac{F_{\text{out}}}{F_{\text{in}}}$

On the other hand, the theoretical or **ideal mechanical advantage**, **IMA**, is based only on the geometry of the system and does not take frictional effects into account.

or IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$$

Because no machine is perfect and because you will always get out less work than you put in, you need to consider the efficiency of the machine that you are using. The more efficient the machine, the greater work output you will get for your work input. The efficiency will always be less than 100%.

Vocabulary Efficiency: The ratio of the work output to the work input.

efficiency =
$$\frac{\text{work output}}{\text{work input}} = \frac{F_{\text{out}} \Delta d_{\text{out}}}{F_{\text{in}} \Delta d_{\text{in}}} = \frac{\text{AMA}}{\text{IMA}}$$

Efficiency has no units and is usually expressed as a percent.

Solved Examples

A crate of bananas weighing 3000. N is shipped from South America to New Example 8: York, where it is unloaded by a dock worker who lifts the crate by pulling with a force of 200. N on the rope of a pulley system. What is the actual mechanical advantage of the pulley system?

Given:
$$F_{\text{out}} = 3000. \text{ N}$$

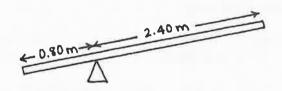
 $F_{\text{in}} = 200. \text{ N}$

Unknown: AMA = ?
Original equation: AMA =
$$\frac{F_{\text{out}}}{F_{\text{in}}}$$

Solve: AMA =
$$\frac{F_{\text{out}}}{F_{\text{in}}} = \frac{3000. \text{ N}}{200. \text{ N}} = 15.0$$

The pulley exerts 15.0 times more force on the crate than the dock worker exerts to pull the rope. Notice that mechanical advantage has no units.

Example 9: Two clowns, of mass 50.0 kg and 70.0 kg respectively, are in a circus act performing a stunt with a trampoline and a seesaw. The smaller clown stands on the lower end of the seesaw while the larger clown jumps from the trampoline onto the raised side of the seesaw, propelling his friend into the air. a) what is the ideal mechanical advantage of the seesaw? b) If the larger clown exerts a force of 850. N on the seesaw as he jumps, how much force is exerted on the smaller clown?



a. The seesaw acts as a lever with the fulcrum 0.80 m from the left side. The ideal mechanical advantage is found by comparing the two distances.

Given:
$$\Delta d_{\text{in}} = 2.40 \text{ m}$$

 $\Delta d_{\text{out}} = 0.80 \text{ m}$

Unknown: IMA = ?
Original equation: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$$

Solve: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \frac{2.40 \text{ m}}{0.80 \text{ m}} = 3.0$$

b. To answer this question, assume that the seesaw is 100% efficient and the work out equals the work in (which is highly unlikely!).

Solve:
$$F_{\text{out}} = \frac{F_{\text{in}} \Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \frac{(850. \text{ N})(2.40 \text{ m})}{0.80 \text{ m}} = 2550 \text{ N}$$

Example 10: A jackscrew with a handle 30.0 cm long is used to lift a car sitting on the jack. The car rises 2.0 cm for every full turn of the handle. What is the ideal mechanical advantage of the jack?

Solution: For a screw, IMA = $\frac{\Delta d_{\rm in}}{\Delta d_{\rm out}} = \frac{2\pi r}{\Delta h}$ where $2\pi r$ is the circumference of the circle through which the handle turns, and height, Δh , refers to the amount the jack (and hence the automobile) is raised.

Given:
$$r = 30.0 \text{ cm}$$
 Unknown: IMA = ?
$$\Delta h = 2.0 \text{ cm}$$
 Original equation: IMA = $\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$

Solve: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \frac{2\pi r}{\Delta h} = \frac{2\pi (30.0 \text{ cm})}{2.0 \text{ cm}} = 94$$

Example 11: Jack and Jill went up the hill to fetch a pail of water. At the well, Jill used a force of 20.0 N to turn a crank handle of radius 0.400 m that rotated an axle of radius 0.100 m, so she could raise a 60.0-N bucket of water. a) What is the ideal mechanical advantage of the wheel? b) What is the actual mechanical advantage of the wheel? c) What is the efficiency of the wheel?

Solution: Since the crank handle and the axle both turn in a circle, $\Delta d_{\rm in} = 2\pi r_{\rm c}$ (where $r_{\rm c}$ is the radius of the crank handle) and $\Delta d_{\rm out} = 2\pi r_{\rm a}$ (where $r_{\rm a}$ is the radius of the axle).

Solve: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \frac{2\pi r_{\text{c}}}{2\pi r_{\text{a}}} = \frac{2\pi (0.400 \text{ m})}{2\pi (0.100 \text{ m})} = 4.00$$

b. The force on the bucket of water is $F_{\rm out}$ and the force exerted by Jill is $F_{\rm in}$.

Unknown: Eff = ?
Original equation: Eff =
$$\frac{AMA}{IMA}$$

Solve: Eff =
$$\frac{AMA}{IMA} = \frac{3.00}{4.00} = 0.750 = 75.0\%$$

Clyde, a stubborn 3500-N mule, refuses to walk into the barn, so Farmer Example 12: MacDonald must drag him up a 5.0-m ramp to his stall, which stands 0.50 m above ground level. a) What is the ideal mechanical advantage of the ramp? b) If Farmer MacDonald needs to exert a 450-N force on the mule to drag him up the ramp with a constant speed, what is the actual mechanical advantage of the ramp? c) What is the efficiency of the ramp?

Solution: For a ramp, ramp length is Δd_{in} and ramp height is Δd_{out} .

a. Given:
$$\Delta d_{\text{in}} = 5.0 \text{ m}$$

 $\Delta d_{\text{out}} = 0.50 \text{ m}$

Unknown: IMA = ? Original equation: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}}$$

Solve: IMA =
$$\frac{\Delta d_{\text{in}}}{\Delta d_{\text{out}}} = \frac{5.0 \text{ m}}{0.50 \text{ m}} = 10.$$

b. Given:
$$F_{\text{out}} = 3500 \text{ N}$$

 $F_{\text{in}} = 450 \text{ N}$

Unknown: AMA = ?
Original equation: AMA =
$$\frac{F_{\text{out}}}{F_{\text{in}}}$$

Solve: AMA =
$$\frac{F_{\text{out}}}{F_{\text{in}}} = \frac{3500 \text{ N}}{450 \text{ N}} = 7.8$$

Unknown: Eff = ?
Original equation: Eff =
$$\frac{AMA}{IMA}$$

Solve: Eff =
$$\frac{AMA}{IMA} = \frac{7.8}{10} = 0.78 = 78\%$$

Practice Exercises

Cathy, a 460-N actress playing Peter Pan, is hoisted above the stage in order to Exercise 11: "fly" by a stagehand pulling with a force of 60. N on a rope wrapped around a pulley system. What is the actual mechanical advantage of the pulley system?

Answer: _

Exercise 12: A windmill uses sails blown by the wind to turn an axle that allows a grindstone to grind corn into meal with a force of 90. N. The windmill has sails of radius 6.0 m blown by a wind that exerts a force of 15 N on the sails, and the axle of the grindstone has a radius of 0.50 m. a) What is the ideal mechanical advantage of the wheel? b) What is the actual mechanical advantage of the wheel? c) What is the efficiency of the wheel? Answer: a_{\bullet} Answer: b. ___ Answer: c. -Winnie, a waitress, holds in one hand a 5.0-N tray stacked with twelve 3.5-N Exercise 13: dishes. The length of her arm from her hand to her elbow is 30.0 cm and her biceps muscle exerts a force 5.0 cm from her elbow, which acts as a fulcrum. How much force must her biceps exert to allow her to hold the tray? Ċm Exercise 14: When building the pyramids, the ancient Egyptians were able to raise large stones to very great heights by using inclines. If an incline has an ideal mechanical advantage of 4.00 and the pyramid is 15.0 m tall, how much of an angle would the incline need in order for the Egyptian builder to reach the top?

Exercise 15: The Ramseys are moving to a new town, so they have called in the ACME moving company to take care of their furniture. Debbie, one of the movers, slides the Ramseys' 2200-N china cabinet up a 6.0-m-long ramp to the moving van, which stands 1.0 m off the ground. a) What is the ideal mechanical advantage of the incline? b) If Debbie must exert a 500.-N force to move the china cabinet up the ramp with a constant speed, what is the actual mechanical advantage of the ramp? c) What is the efficiency of the ramp?

Answer: a	
Answer: b.	
Answer: c.	

Additional Exercises

- A-1: On a ski weekend in Colorado, Bob, whose mass is 75.0 kg, skis down a hill that is inclined at an angle of 15.0° to the horizontal and has a vertical rise of 25.0 m. How much work is done by gravity on Bob as he goes down the hill?
- A-2: A pile driver is a device used to drive stakes into the ground. While building a fence, Adam drops a pile driver of mass 3000. kg through a vertical distance of 8.0 m. The pile driver is opposed by a resisting force of 5.0×10^6 N. How far is the stake driven into the ground on the first stroke?
- A-3: At Six Flags New England in Agawam, Massachusetts, a ride called the Cyclone is a giant roller coaster that ascends a 34.1-m hill and then drops 21.9 m before ascending the next hill. The train of cars has a mass of 4727 kg. a) How much work is required to get an empty train of cars from the ground to the top of the first hill? b) What power must be generated to bring the train to the top of the first hill in 30.0 s? c) How much PE is converted into KE from the top of the first hill to the bottom of the 21.9-m drop?
- A-4: A flea gains 1.0×10^{-7} J of PE jumping up to a height of 0.030 m from a dog's back. What is the mass of the flea?

- A-5: At target practice, Diana holds her bow and pulls the arrow back a distance of 0.30 m by exerting an average force of 40.0 N. What is the potential energy stored in the bow the moment before the arrow is released?
- A-6: The coyote, whose mass is 20.0 kg, is chasing the roadrunner when the coyote accidentally runs off the edge of a cliff and plummets to the ground 30.0 m below. What force does the ground exert on the coyote as he makes a coyote-shaped dent 0.420 m deep in the ground?
- A-7: A 0.080-kg robin, perched on a power line 6.0 m above the ground, swoops down to snatch a worm from the ground and then returns to an 8.0-m-high tree branch with his catch. a) By how much did the bird's PE increase in its trip from the power line to the tree branch? b) How would your answer have changed if the bird had flown around a bit before landing on the tree branch?
- A-8: Blackie, a cat whose mass is 5.45 kg, is napping on top of the refrigerator when he rolls over and falls. Blackie has a KE of 85.5 J just before he lands on his feet on the floor. How tall is the refrigerator?
- A-9: Calories measure energy we get from food, and one dietary Calorie is equal to 4187 J. The average food energy intake for human beings is 2000. Calories/day. Assume you have a mass of 55.0 kg and you want to burn off all the Calories you consume in one day. How high a mountain would you have to climb to do so? (Note: This calculation ignores the large amount of energy the body continually loses to heat.)
- A-10: From a height of 2.15 m above the floor of Boston's Fleet Center, forward Paul Pierce tosses a shot straight up next to the basketball hoop with a KE of 5.40 J. If his regulation-size basketball has a mass of 0.600 kg, will his shot go as high as the 3.04-m hoop? Use the law of conservation of energy.
- A-11: Mr. Macintosh, a computer technician, uses a screwdriver with a handle of radius 1.2 cm to remove a screw in the back of a computer. The screw moves out 0.20 cm on each complete turn. What is the ideal mechanical advantage of the screwdriver?
- A-12: Tom's favorite pastime is fishing. a) How much work is required for Tom to reel in a 10.0-kg bluefish from the water's surface to the deck of a fishing boat, 5.20 m above the water, if the reel of his fishing pole is 85.0% efficient? b) If Tom applies a force of 15 N to the reel's crank handle, what is the actual mechanical advantage of the fishing pole? c) What is the ideal mechanical advantage of the fishing pole?
- A-13: A nutcracker 16 cm long is used to crack open a Brazil nut that is placed 12 cm from where your hand is squeezing the nutcracker. What is the ideal mechanical advantage of the nutcracker?

Challenge Exercises for Further Study

- B-1: A 5.00-N salmon swims 20.0 m upstream against a current that provides a resistance of 1.50 N. This portion of the stream rises at an angle of 10.0° with respect to the horizontal. a) How much work is done by the salmon against the current? b) What is the gain in PE by the salmon? c) What is the total work that must be done by the salmon? d) If the salmon takes 40.0 s to swim the distance, what power does it exert in doing so?
- A 30-kg shopping cart full of groceries sitting at the top of a 2.0-m hill begins B-2: to roll until it hits a stump at the bottom of the hill. Upon impact, a 0.25-kg can of peaches flies horizontally out of the shopping cart and hits a parked car with an average force of 490 N. How deep a dent is made in the car?
- B-3: Using her snowmobile, Midge pulls a 60.0-kg skier up a ski slope inclined at an angle of 12.0° to the horizontal. The snowmobile exerts a force of 200. N parallel to the hill. If the coefficient of friction between the skis and the snow is 0.120, how fast is the skier moving after he has been pulled for 100.0 m starting from rest? (Ignore the effects of the static friction that must be overcome to initially start him in motion.) Use the law of conservation of energy.
- Jose, whose mass is 45.0 kg, is riding his 5.0-kg skateboard down the sidewalk B-4: with a constant speed of 6.0 m/s when he rolls across a 10.0-m-long patch of sand on the pavement. The sand provides a force of friction of 6.0 N. What is Jose's speed as he emerges from the sandy section?
- B-5: Eben lifts an engine out of his Volkswagen with the help of a winch that allows him to raise the engine 0.020 m for every 0.90 m he pulls on the cable. Eben expends 1000. J of energy to lift the 800.-N engine 0.50 m. a) What is the efficiency of the winch? b) What is the ideal mechanical advantage of the winch? c) What is the actual mechanical advantage of the winch? d) What force does Eben exert to lift the engine?

