6 Circular Motion

6-1 Centripetal Acceleration and Force

Period, Frequency, and Speed

Vocabulary **Period:** The time it takes for one full rotation or revolution of an object.

Vocabulary **Frequency:** The number of rotations or revolutions per unit time.

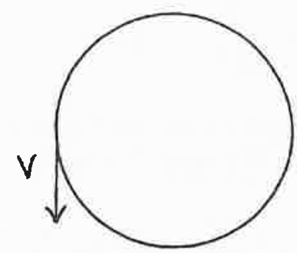
Period and frequency are reciprocals of each other. In other words,

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Since period is a measure of time, its SI unit is the **second**, while the unit for frequency is the reciprocal of this, or 1/second. Another way of writing 1/second is with the unit **hertz (Hz)**.

When an object spins in a circle, the distance it travels in one revolution is the circumference of the circle, $2\pi r$. The time it takes for one revolution is the period, T. Therefore,

$$\textbf{speed} = \frac{\mathbf{2\pi(radius)}}{\textbf{period}} \quad \text{or} \quad v = \frac{2\pi r}{T}$$



where v is called the **linear** or **tangential speed** because at any given time, the velocity is tangent to the circle as shown in the diagram. Although the velocity is constant in magnitude (speed), it is always changing direction.

Centripetal Acceleration and Centripetal Force

An object can move around in a circle with a constant speed yet still be accelerating because its direction is constantly changing. This acceleration, which is always directed in toward the center of the circle, is called **centripetal acceleration**. The magnitude of this acceleration is written as

$$\textbf{centripetal acceleration} = \frac{\textbf{(linear speed)}^2}{\textbf{radius}} \quad \text{or} \quad a_c = \frac{v^2}{r}$$

If a mass is being accelerated toward the center of a circle, it must be acted upon by an unbalanced force that gives it this acceleration. This force, called the **centripetal force**, is always directed inward toward the center of the circle. The magnitude of this force is written as

centripetal force = (mass)(centripetal acceleration)

$$\text{or} \quad F_c = ma_c = \frac{mv^2}{r}$$

The units for centripetal acceleration and centripetal force are m/s^2 and N, respectively.

Solved Examples

Example 1: After closing a deal with a client, Kent leans back in his swivel chair and spins around with a frequency of 0.5 Hz. What is Kent's period of spin?

Given: $f = 0.5$ Hz

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{0.5 \text{ Hz}} = \mathbf{2\ s}$

Example 2: Curtis' favorite disco record has a scratch 12 cm from the center that makes the record skip 45 times each minute. What is the linear speed of the scratch as it turns?

Solution: The record makes 45 revolutions every 60. seconds, so find the period of the record first.

$$T = \frac{60.\text{ s}}{45 \text{ rev}} = 1.3 \text{ s}$$

Given: $r = 12$ cm
$T = 1.3$ s

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(12 \text{ cm})}{1.3 \text{ s}} = \mathbf{58\ cm/s}$

Example 3: Missy's favorite ride at the Topsfield Fair is the rotor, which has a radius of 4.0 m. The ride takes 2.0 s to make one full revolution.
a) What is Missy's linear speed on the rotor?
b) What is Missy's centripetal acceleration on the rotor?

Solution: The ride takes 2.0 s to make one full revolution, so the period is 2.0 s.

a. *Given:* $r = 4.0\text{ m}$
$T = 2.0\text{ s}$

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(4.0\text{ m})}{2.0\text{ s}} = \mathbf{13\text{ m/s}}$

b. *Given:* $v = 13\text{ m/s}$
$r = 4.0\text{ m}$

Unknown: $a_c = ?$
Original equation: $a_c = \frac{v^2}{r}$

Solve: $a_c = \frac{v^2}{r} = \frac{(13\text{ m/s})^2}{4.0\text{ m}} = \mathbf{42\text{ m/s}^2}$

Example 4: Captain Chip, the pilot of a 60 500-kg jet plane, is told that he must remain in a holding pattern over the airport until it is his turn to land. If Captain Chip flies his plane in a circle whose radius is 50.0 km once every 30.0 min, what centripetal force must the air exert against the wings to keep the plane moving in a circle?

Solution: First, convert km to m and min to s.

$50.0\text{ km} = 5.00 \times 10^4\text{ m}$ $\quad$ $30.0\text{ min} = 1.80 \times 10^3\text{ s}$

Before solving for the centripetal force, find the speed of the airplane.

Given: $T = 1.80 \times 10^3\text{ s}$
$r = 5.00 \times 10^4\text{ m}$

Unknown: $v = ?$
Original equation: $v = \frac{2\pi r}{T}$

Solve: $v = \frac{2\pi r}{T} = \frac{2\pi(5.00 \times 10^4\text{ m})}{1.80 \times 10^3\text{ s}} = \mathbf{175\text{ m/s}}$

Use this speed to solve for the centripetal force.

Given: $m = 60\,500\text{ kg}$
$v = 175\text{ m/s}$
$r = 5.00 \times 10^4\text{ m}$

Unknown: $F_c = ?$
Original equation: $F_c = \frac{mv^2}{r}$

Solve: $F_c = \frac{mv^2}{r} = \frac{(60\,500\text{ kg})(175\text{ m/s})^2}{5.00 \times 10^4\text{ m}} = \mathbf{3.71 \times 10^4\text{ N}}$

Practice Exercises

Exercise 1: Marianne puts her favorite Backstreet Boys disc in her CD player. If it spins with a frequency of 1800 revolutions per minute, what is the period of spin of the compact disc?

Answer: ____________________

Exercise 2: Hamlet, a hamster, runs on his exercise wheel, which turns around once every 0.5 s. What is the frequency of the wheel?

Answer: ____________________

Exercise 3: A sock stuck to the inside of the clothes dryer spins around the drum once every 2.0 s at a distance of 0.50 m from the center of the drum. a) What is the sock's linear speed? b) If the drum were twice as wide but continued to turn with the same frequency, would the linear speed of a sock stuck to the inside be faster than, slower than, or the same speed as your answer to part a?

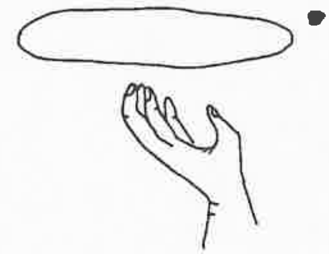
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: What is the radius of an automobile tire that turns with a frequency of 11 Hz and has a linear speed of 20.0 m/s?

Answer: ____________________

Exercise 5: Luigi twirls a round piece of pizza dough overhead with a frequency of 60 revolutions per minute. a) Find the linear speed of a stray piece of pepperoni stuck on the dough 10. cm from the pizza's center. b) In what direction will the pepperoni move if it flies off while the pizza is spinning? Explain the reason for your answer.



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 6: Earth turns on its axis approximately once every 24 hours. The radius of Earth is 6.38×10^6 m. a) If some astronomical catastrophe suddenly brought Earth to a screeching halt (a physical impossibility as far as we know), with what speed would Earth's inhabitants who live at the equator go flying off Earth's surface? b) Because Earth is solid, it must turn with the same frequency everywhere on its surface. Compare your linear speed at the equator to your linear speed while standing near one of the poles.

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 7: Jessica is riding on a merry-go-round on an outer horse that sits at a distance of 8.0 m from the center of the ride. Jessica's sister, Julie, is on an inner horse located 6.0 m from the ride's center. The merry-go-round turns around once every 40.0 s. a) Explain which girl is moving with the greater linear speed. b) What is the centripetal acceleration of Julie and her horse?

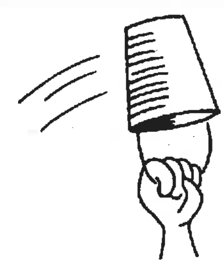
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 8: A cement mixer of radius 2.5 m turns with a frequency of 0.020 Hz. What is the centripetal acceleration of a small piece of dried cement stuck to the inside wall of the mixer?

Answer: ____________________

Exercise 9: A popular trick of many physics teachers is to swing a pail of water around in a vertical circle fast enough so that the water doesn't spill out when the pail is upside down. If Mr. Lowell's arm is 0.60 m long, what is the minimum speed with which he can swing the pail so that the water doesn't spill out at the top of the path?



Answer: ____________________

Exercise 10: To test their stamina, astronauts are subjected to many rigorous physical tests before they fly in space. One such test involves spinning the astronauts in a device called a *centrifuge* that subjects them to accelerations far greater than gravity. With what linear speed would an astronaut have to spin in order to experience an acceleration of 3 *g*'s at a radius of 10.0 m? (1 g = 10.0 m/s^2)

Answer: ____________________

Exercise 11: At the Fermilab particle accelerator in Batavia, Illinois, protons are accelerated by electromagnets around a circular chamber of 1.00-km radius to speeds near the speed of light before colliding with a target to produce enormous amounts of energy. If a proton is traveling at 10% the speed of light, how much centripetal force is exerted by the electromagnets? (Hint: The speed of light is 3.00×10^8 m/s, $m_p = 1.67 \times 10^{-27}$ kg)

Answer: ____________________

Exercise 12: Roxanne is making a strawberry milkshake in her blender. A tiny, 0.0050-kg strawberry is rapidly spun around the inside of the container with a speed of 14.0 m/s, held by a centripetal force of 10.0 N. What is the radius of the blender at this location?

Answer: ____________________

6-2 Torque

Vocabulary **Torque:** A measurement of the tendency of a force to produce a rotation about an axis.

torque = perpendicular force × lever arm or $\tau = F \times d$

The lever arm, *d*, is the distance from the pivot point, or fulcrum, to the point where the component of the force perpendicular to the lever arm is being exerted. The longer the lever arm, the larger the torque. This is why it is easier to loosen a tight screw with a long wrench than with your hand or a short pair of tweezers.

If a torque causes a counterclockwise rotation of an object around the fulcrum, it is positive. If the torque causes a clockwise rotation of an object around the fulcrum, it is negative. This convention works even if the object remains balanced and the torques just *attempt* to cause a rotation.

The SI unit for torque is the **newton·meter (N·m)**. However, unlike work, which is measured in the same unit, torque is not a form of energy and is not equivalent to a joule.

In most of the exercises in this book, all the torques are balanced. For example, if two people are sitting on either side of a seesaw and they want to remain level, they can position themselves so that all the torques on one side of the seesaw equal all the torques on the other side. The total torque on a system equals the sum of all the individual torques, or

$$\tau = (F_1 \times d_1) + (F_2 \times d_2) + \dots$$

The ... means that there may be more than only two torques acting on a system at any one time. Keep in mind that when an object is balanced, all the torques must also balance. Therefore, the total torque, τ, is zero.

Vocabulary **Center of Gravity:** The point on any object that acts like the place at which all the weight is concentrated.

The weight of an object, which acts as if it is concentrated at the center of gravity, is one of the forces that can cause it to rotate. The weight produces a torque if the object is not supported at its center of gravity.

Solved Examples

Example 5: Ned tightens a bolt in his car engine by exerting 12 N of force on his wrench at a distance of 0.40 m from the fulcrum. How much torque must Ned produce to turn the bolt?

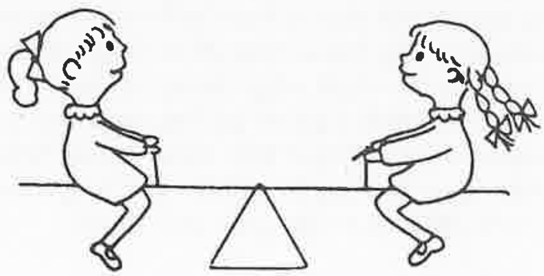
Given: $F = 12\text{ N}$
$d = 0.40\text{ m}$

Unknown: $\tau = ?$
Original equation: $\tau = F \times d$

Solve: $\tau = F \times d = (12\text{ N})(0.40\text{ m}) = \mathbf{4.8\ N{\cdot}m}$

Example 6: Mabel and Maude are seesawing on the school playground and decide to see if they can move to the correct location to make the seesaw balance. Mabel weighs 400. N and she sits 2.00 m from the fulcrum of the seesaw. Where should 450.-N Maude sit to balance the seesaw?

Solution: It helps to draw a diagram of the situation to allow yourself to visualize what is happening.



Given: $F_1 = 400.\ \text{N}$
$F_2 = 450.\ \text{N}$
$d_1 = 2.00\ \text{m}$

Unknown: $d_2 = ?$
Original equation:
$\tau = (F_1 \times d_1) + (F_2 \times d_2)$

Solve: If 400.-N Mabel makes the seesaw turn in a counterclockwise direction, then 450.-N Maude makes the seesaw turn in a clockwise direction. Therefore, $\tau = (F_1 \times d_1) + -(F_2 \times d_2)$. If the seesaw is balanced, then $\tau = 0$ and the equation becomes $\tau = (F_1 \times d_1) + -(F_2 \times d_2) = 0$, or $(F_1 \times d_1) = (F_2 \times d_2)$. Therefore,

$$d_2 = \frac{(F_1 \times d_1)}{F_2} = \frac{(400.\ \text{N})(2.00\ \text{m})}{450.\ \text{N}} = \mathbf{1.78\ m}\ \text{from the fulcrum.}$$

Practice Exercises

Exercise 13: A water faucet is turned on when a force of 2.0 N is exerted on the handle, at a distance of 0.060 m from the pivot point. How much torque must be produced to turn the handle?

Answer: ____________________

Exercise 14: Nancy, whose mass is 60.0 kg, is working at a construction site and she sits down for a bite to eat at noon. If Nancy sits on the very end of a 3.00-m-long plank pivoted in the middle on a saw horse, how much torque must her co-worker provide on the other end of the plank in order to keep Nancy from falling on the ground?

Answer: ____________________

Exercise 15: Barry carries his tray of food to his favorite cafeteria table for lunch. The 0.50-m-long tray has a mass of 0.20 kg and holds a 0.40-kg plate of food 0.20 m from the right edge. Barry holds the tray by the left edge with one hand, using his thumb as the fulcrum, and pushes up 0.10 m from the fulcrum with his finger tips. How much upward force must his finger tips exert to keep the tray level? b) How might Barry make the tray easier to carry if he still chooses to use only one hand?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 16: Soon-Yi is building a mobile to hang over her baby's crib. She hangs a 0.020-kg toy sailboat 0.010 m from the left end and a 0.015-kg toy truck 0.20 m from the right end of a bar 0.50 m long. If the lever arm itself has negligible mass, where must the support string be placed so that the arm balances?

Answer: ______________________

Exercise 17: Orin and Anita, two paramedics, rush a 60.0-kg man from the scene of an accident to a waiting ambulance, carrying him on a uniform 3.00-kg stretcher held by the ends. The stretcher is 2.60 m long and the man's center of mass is 1.00 m from Anita. How much force must Orin and Anita each exert to keep the man horizontal?

Answer: ______________________

6-3 Moment of Inertia and Angular Momentum

Vocabulary **Moment of Inertia:** The resistance of an object to changes in its rotational motion.

The equation for the moment of inertia varies depending upon the shape of the rotating object. For an object rotating around an axis at a distance *r*,

$$\textbf{moment of inertia} = \textbf{(mass)(radius)}^2 \qquad \text{or} \qquad I = mr^2$$

The SI unit for moment of inertia is the **kilogram·meter squared (kg·m²)**.

Other moments of inertia can be found in your textbook, and are summarized as follows.

hoop rotating about its center: $I = mr^2$
hoop rotating about its diameter: $I = \left(\frac{1}{2}\right)mr^2$
solid cylinder: $I = \left(\frac{1}{2}\right)mr^2$
stick rotating about its center of gravity: $I = \left(\frac{1}{12}\right)m\ell^2$
stick rotating about its end: $I = \left(\frac{1}{3}\right)m\ell^2$
solid sphere rotating about its center of gravity: $I = \left(\frac{2}{5}\right)mr^2$

Newton's first law says that inertia is the tendency of an object to stay at rest or remain in motion in a straight line with a constant speed unless acted upon by an unbalanced force. Similarly, an object that is rotating tends to continue spinning at a constant rate unless an unbalanced force acts to alter that rotation. This is called the rotational inertia.

Think of moment of inertia as being the rotational equivalent of the term "mass." Just as inertia is greater for a greater mass, rotational inertia is greater for a greater moment of inertia.

Vocabulary **Angular Momentum:** The measure of how difficult it is to stop a rotating object.

$$\textbf{angular momentum} = \textbf{(mass)(velocity)(radius)} \qquad \text{or} \qquad L = mvr$$

The SI unit for angular momentum is the **kilogram·meter squared per second (kg·m²/s)**.

Think of angular momentum as being the rotational equivalent of the term *linear momentum.* Just as linear momentum is the product of the mass and the velocity, angular momentum is the product of the mass and the velocity for an object rotating at a distance *r* from the axis.

Momentum is conserved when no outside forces are acting. Similarly, angular momentum is conserved when no outside torques are acting. A spinning ice skater has angular momentum. When the skater pulls her arms in (decreasing her radius of spin), she spins faster (increasing her velocity). Doing so conserves her angular momentum.

Solved Examples

Example 7: On the Wheel of Fortune game show, a contestant spins the 15.0-kg wheel that has a radius of 1.40 m. What is the moment of inertia of this disk-shaped wheel?

Solution: A disk is a thin cylinder, so the moment of inertia of a disk is the same as that of a cylinder.

Given: $m = 15.0\text{ kg}$
$r = 1.40\text{ m}$

Unknown: $I = ?$
Original equation: $I = \left(\frac{1}{2}\right)mr^2$

Solve: $I = \left(\frac{1}{2}\right)(15.0\text{ kg})(1.40\text{ m})^2 = \mathbf{14.7\ kg{\cdot}m^2}$

Example 8: Trish is twirling her 0.60-m majorette's baton that has a mass of 0.40 kg. What is the moment of inertia of the baton as it spins about its center of gravity?

Given: $m = 0.40\text{ kg}$
$\ell = 0.60\text{ m}$

Unknown: $I = ?$
Original equation: $I = \left(\frac{1}{2}\right)m\ell^2$

Solve: $I = \left(\frac{1}{2}\right)m\ell^2 = \left(\frac{1}{2}\right)(0.40\text{ kg})(0.60\text{ m})^2 = \mathbf{0.072\ kg{\cdot}m^2}$

Example 9: At Wellesley College in Massachusetts there is a favorite tradition called hoop rolling. In their caps and gowns, seniors roll wooden hoops in a race in which the winner is said to be the first in the class to marry. Hilary rolls her 0.2-kg hoop across the finish line. The moment of inertia of the hoop is $0.032\text{ kg}{\cdot}\text{m}^2$. What is the radius of the hoop?

Given: $m = 0.2\text{ kg}$
$I = 0.032\text{ kg}{\cdot}\text{m}^2$

Unknown: $r = ?$
Original equation: $I = mr^2$

Solve: $r = \sqrt{\frac{I}{m}} = \sqrt{\frac{0.032\text{ kg}{\cdot}\text{m}^2}{0.2\text{ kg}}} = \sqrt{0.16\text{ m}^2} = \mathbf{0.4\ m}$

Example 10: Jupiter orbits the sun with a speed of 2079 m/s at an average distance of 71 398 000 m. a) If Jupiter has a mass of 1.90×10^{27} kg, what is its angular momentum as it orbits?

Given: $m = 1.90 \times 10^{27}$ kg
$v = 2079$ m/s
$r = 71\,398\,000$ m

Unknown: $L = ?$
Original equation: $L = mvr$

Solve: $L = mvr = (1.90 \times 10^{27}\text{ kg})(2079\text{ m/s})(71\,398\,000\text{ m}) = \mathbf{2.82 \times 10^{38}\ kg{\cdot}m^2/s}$

Practice Exercises

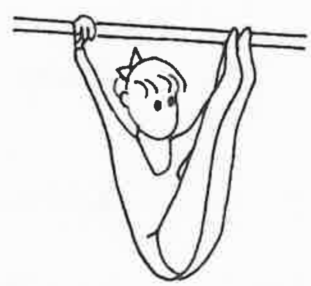
Exercise 18: Veanna is in Las Vegas waiting for her number to be called at the roulette wheel, a large 3.0-kg disk of radius 0.60 m. What is the moment of inertia of the wheel?

Answer: ____________________

Exercise 19: Earth has a mass of 5.98×10^{24} kg and a radius of 6.38×10^{6} m. What is the moment of inertia of Earth as it turns on its axis?

Answer: ____________________

Exercise 20: Olga, the 50.0-kg gymnast, swings her 1.6-m-long body around a bar by her outstretched arms. a) What is Olga's moment of inertia? b) If Olga were to pull in her legs, thereby cutting her body length in half, how would this change her moment of inertia? (Assume her mass is evenly distributed all along her body.)



Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 21: Priya removes her 0.012-kg, 0.60-cm-diameter wedding band and spins it on the coffee table on its edge. What is the moment of inertia of the ring?

Answer: ____________________

Exercise 22: Hickory dickory dock, the 20.0-g mouse ran up the clock, and took turns riding on the 0.20-m-long second hand, the 0.20-m-long minute hand, and the 0.10-m-long hour hand. What was the angular momentum of the mouse on each of the three hands?

Answer: ____________________

Answer: ____________________

Answer: ____________________

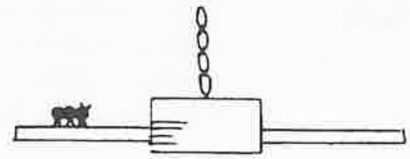
Exercise 23: In a physics experiment, Ingrid, the ice skater, spins around in the rink at 1.2 m/s with each of her arms stretched out 0.70 m from the center of her body. In each hand she holds a 1.0-kg mass. If angular momentum is conserved, how fast will Ingrid begin to spin if she pulls her arms to a position 0.15 m from the center of her body?

Answer: ____________________

Additional Exercises

A-1: In the Biblical tale of David and Goliath, the giant is slain when David hits him with a rock that he has spun around overhead in a sling. If the rock is spun with a frequency of 100 revolutions per minute, what is the rock's period?

A-2: Ashton the ant is crawling on the still blade of a ceiling fan when the fan is turned on, causing Ashton to go for a ride. If Ashton sits on the fan blade at a distance of 0.80 m from the center of the fan and turns with a frequency of 1.2 Hz, a) how fast does Ashton spin? b) If Ashton slips off the spinning fan, describe the path he will take.



A-3: In "Rumpelstiltskin," the miller's daughter is spinning straw into gold on a spinning wheel that turns at a speed of 7.5 m/s, making one revolution every 0.50 s. How long is a strand of gold that makes one complete turn around the wheel?

A-4: A 3.20-kg hawk circles overhead in search of prey. a) If the hawk circles once every 10.0 s in a circle 12.0 m in radius, what is the linear speed of the hawk? b) What centripetal force allows him to remain in a circle? c) What is providing the centripetal force?

A-5: Sasha's favorite ride at the fair is the Ferris wheel that has a radius of 7.0 m. a) If the ride takes 20.0 s to make one full revolution, what is the linear speed of the wheel? b) What centripetal force will the ride exert on Sasha's 50.0-kg body? c) Does Sasha feel as if she is being pulled in or out by the ride? d) Explain the difference between what she feels and what is really happening at the top and bottom of the wheel.

A-6: In order for Sasha (in A-5) to feel weightless at the top of the ride, a) at what linear speed must the Ferris wheel turn? b) At this speed, how much will she appear to weigh at the bottom of the Ferris wheel?

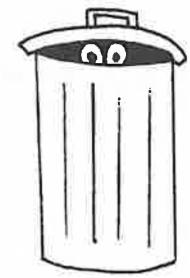
A-7: Earth orbits the sun approximately once every 365.25 days at an average distance of about 1.5×10^{11} m. The mass of Earth is 5.98×10^{24}kg. a) What is the centripetal acceleration of Earth? b) What is the centripetal force of the sun on Earth? c) What is the centripetal force of Earth on the sun? d) If this force exists between the sun and Earth, does this mean that Earth is "falling into" the sun? Explain.

A-8: Most doorknobs are placed on the side of the door opposite the hinges instead of in the center of the door. a) Why is this so? b) If a torque of 1.2 N·m is required to open a door, how much force must be exerted on a doorknob 0.76 m from the hinges compared to a doorknob in the middle of the door, 0.38 m from the hinges?

A-9: Priscilla is working out in the gym with a 2.00-kg mass that she holds in one hand and gradually lifts up and down. a) Will Priscilla find it easier to lift the mass if she pivots her arm at the shoulder or at the elbow? b) If Priscilla's arm is 0.60 m long from her shoulder to her palm and 0.28 m long from her elbow to her palm, how much torque must she produce in each case to lift the weight?

A-10: Leif and Paige are rearranging the heights of their movable bookshelves; they remove one of the 2.00-kg, 0.60-m-long shelves by the two of them holding opposite ends. A 5.00-kg stack of books is piled up on the shelf 0.20 m from Leif. How much force must Leif and Paige each exert to hold the shelf level?

A-11: Brewster hits a 0.30-kg pool ball across the pool table and sinks it in the side pocket. If the pool ball has a radius of 3.5 cm, what is its moment of inertia as it rolls?



A-12: Rocky, a raccoon, squeezes into a 0.60-m-diameter cylindrical trash can to find a late-night snack. However, the can tips over and begins to roll. If Rocky and the can have a combined mass of 40.0 kg, what is the moment of inertia of the system?

A-13: Mieko sharpens a knife on a grinding wheel whose angular momentum is 27 $kg \cdot m^2/s$. The 5.0-kg wheel has a radius of 0.30 m. What is the linear speed of the wheel?

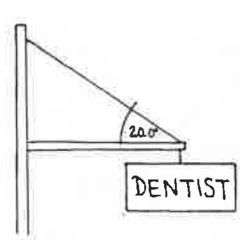
Challenge Problems for Further Study

B-1: The "Bake-a-Lite" Cake Company truck is on its way to deliver a birthday cake for the MacKenzie party when it rounds a curve of radius 20.0 m at a speed of 12 m/s. What coefficient of friction is needed between the cake pan and the truck in order to keep the pan from slipping?

B-2: On his way home from the office, Steven's car rounds an unbanked curve that has a radius of 100 m. If the coefficient of friction between the tires and the road is 0.40, what is the fastest speed at which the car can round this curve without risking an accident?

B-3: Pretending to be Tarzan, 50.0-kg Zach swings from the end of a 5.0-m-long rope attached to a tree branch. The tree branch will break if subjected to a force greater than 750 N. What is the maximum speed with which Zach can swing in order to avoid breaking the branch?

B-4: Hanging in front of the office of Lewis Skeirik, D.D.M., is a sign that weighs 120 N and is suspended at the end of a 0.80-m-long support beam that weighs 10.0 N, as shown. What is the tension in a supporting wire that holds the sign at an angle of 20.0°?



7 Law of Universal Gravitation

7-1 Gravitational Force

Vocabulary **Law of Universal Gravitation:** Every particle attracts every other particle with a force that is proportional to the mass of the particles and inversely proportional to the square of the distance between them.

$$F \propto \frac{mM}{d^2}$$

The sign $\propto$ means "proportional to." To make an equation out of the above situation, insert a quantity called the **universal constant of gravitation, *G*.**

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Now the magnitude of this gravitational force can be represented as

$$\textbf{Force} = \frac{\textbf{(universal constant of gravitation)(mass 1)(mass 2)}}{\textbf{(distance)}^2}$$

or $$F = \frac{GmM}{d^2}$$

Like all other forces, the gravitational force of attraction between two objects is measured in newtons.

Solved Examples

Example 1: The gravitational force of attraction between Earth and the sun is 1.6×10^{23} N. What would this force have been if Earth were twice as massive?

Solution: The gravitational force of attraction between two bodies is proportional to the mass of each of the two bodies. As one mass increases, the gravitational force between the two bodies increases proportionally. Therefore, if Earth's mass were doubled, the gravitational force between the sun and Earth would double as well.

Therefore, $F = 2F_o = 2(1.6 \times 10^{23}\ \mathrm{N}) = \mathbf{3.2 \times 10^{23}\ N}$

Example 2: The gravitational force of attraction between Earth and the sun is 1.6×10^{23} N. What would this gravitational force have been if Earth had formed twice as far away from the sun?

Solution: The gravitational force of attraction between two bodies is inversely proportional to the square of the distance between them. In this case, if the distance is twice as great, the force between Earth and the sun would be 1/4 as much.

$$\text{Therefore, } F \propto \frac{1}{d^2} \quad \text{or} \quad F = \frac{F_o}{4} = \frac{(1.6 \times 10^{23}\text{ N})}{4} = \mathbf{4.0 \times 10^{22}\ N}$$

Example 3: Oliver, whose mass is 65 kg, and Olivia, whose mass is 45 kg, sit 2.0 m apart in their physics classroom. a) What is the force of gravitational attraction between Oliver and Olivia? b) Why don't Oliver and Olivia drift toward each other?

a) *Given:* $m_{\text{Oliver}} = 65$ kg
$M_{\text{Olivia}} = 45$ kg
$d = 2.0$ m
$G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$

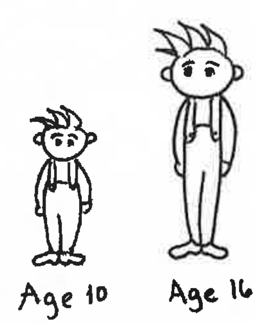
Unknown: $F = ?$
Original equation: $F = \dfrac{GmM}{d^2}$

$$\textit{Solve: } F = \frac{GmM}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(65\text{ kg})(45\text{ kg})}{(2.0\text{ m})^2} = \mathbf{4.9 \times 10^{-8}\ N}$$

b) Because the gravitational force of Earth is much greater than the force Oliver and Olivia exert on each other.

Practice Exercises

Exercise 1: When Royce was 10 years old, he had a mass of 30 kg. By the time he was 16 years old, his mass increased to 60 kg. How much larger is the gravitational force between Royce and Earth at age 16 compared to age 10?



Answer: ____________________

Exercise 2: If John Glenn weighed 640 N on Earth's surface, a) how much would he have weighed if his Mercury spacecraft had (hypothetically) remained at twice the distance from the center of Earth? b) Why is it said that an astronaut is never truly "weightless?"

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 3: Mr. Gewanter, whose mass is 60.0 kg, is doing a physics demonstration in the front of the classroom. a) How much gravitational force does he exert on 55.0-kg Martha in the front row, 1.50 m away? b) How does this compare to what he exerts on 65.0-kg Lester, 4.00 m away in the back row?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 4: Astrologers claim that your personality traits are determined by the positions of the planets in relation to you at birth. Scientists argue that these gravitational effects are so small that they are totally insignificant. Compare the gravitational attraction between you and Mars to the gravitational attraction between you and your 70.0-kg doctor at the moment of your birth, if the doctor stands 0.500 m away. (Note: $M_M = 6.42 \times 10^{23}$ kg, $d_{E \text{ to } M} = 7.83 \times 10^{10}$ m. This is the average distance between Earth and Mars. This distance varies as the two planets orbit the sun.)

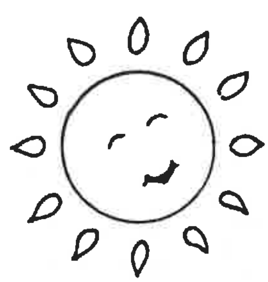
Answer: ____________________

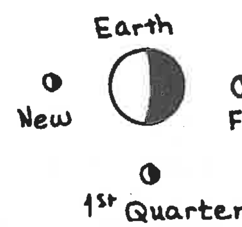
Answer: ____________________

Exercise 5: Our galaxy, the Milky Way, contains approximately 4.0×10^{11} stars with an average mass of 2.0×10^{30} kg each. How far away is the Milky Way from our nearest neighbor, the Andromeda Galaxy, if Andromeda contains roughly the same number of stars and attracts the Milky Way with a gravitational force of 2.4×10^{30} N?

Answer: ______________________

Exercise 6: Tides are created by the gravitational attraction of the sun and moon on Earth. Calculate the net force pulling on Earth during a) a new moon, b) a full moon, c) a first quarter moon. The diagram is intended to help your understanding of the situation but is *not* drawn to scale. ($m_M = 7.35 \times 10^{22}$ kg, $m_E = 5.98 \times 10^{24}$ kg, $m_S = 1.99 \times 10^{30}$ kg, $d_{E-M} = 3.84 \times 10^{8}$ m, $d_{E-S} = 1.50 \times 10^{11}$ m)





Answer: **a.** ______________________

Answer: **b.** ______________________

Answer: **c.** ______________________

7-2 Gravitational Acceleration

You can use the law of universal gravitation to find the gravitational acceleration, g, of any body if you know that body's mass and radius. For example, let's look at the situation on Earth. The weight of an object on Earth's surface is equal to the gravitational force between that object and Earth:

$$mg = \frac{GmM}{d^2}$$

The m on the left represents the mass of an object, such as a human being. The m on the right side of the equation stands for this same mass, so the term cancels out of the equation. The M on the right represents the mass of Earth or other celestial body on which the person is standing. The d in the denominator is equal to the radius of the celestial body. So the equation becomes

$$g = \frac{GM}{d^2}$$

In this equation, g is the acceleration due to gravity on the celestial body in question. On Earth you already know that this value is 10.0 m/s^2.

Solved Examples

Example 4: Temba is standing in the lunch line 6.38×10^6 m from the center of Earth. Earth's mass is 5.98×10^{24} kg. a) What is the acceleration due to gravity? b) When Temba eats his lunch and his mass increases, does this change the acceleration due to gravity?

a. *Given:* $M = 5.98 \times 10^{24}$ kg
$d = 6.38 \times 10^6$ m
$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Unknown: $g = ?$
Original equation: $g = \frac{GM}{d^2}$

Solve: $$g = \frac{GM}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(6.38 \times 10^6\ \text{m})^2} = \mathbf{9.80\ m/s^2}$$

b. No, his acceleration due to gravity does not change because it is not dependent on his mass.

Example 5: The sun has a mass that is 333 000 times Earth's mass and a radius 109 times Earth's radius. What is the acceleration due to gravity on the sun?

Solution: One way to solve this exercise is to actually multiply the given values by the mass and radius of Earth. However, there is an easier and much neater way to come up with the correct answer. By working with ratios, you can find an answer without any information about Earth.

Given: $M_S = 333\,000\ M_E$
$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$
$d_s = 109\ d_E$

Unknown: $g = ?$
Original equation: $g = \dfrac{GM}{d^2}$

Solve: Set up the above equation as a ratio of sun to Earth before substituting numbers.

$$\frac{g_S}{g_E} = \frac{\dfrac{GM_S}{{d_S}^2}}{\dfrac{GM_E}{{d_E}^2}}$$

Simplifying gives
$$\frac{g_S}{g_E} = \frac{M_S {d_E}^2}{M_E {d_S}^2} = \frac{(333\,000\ M_E)({d_E}^2)}{(M_E)(109\,d_E)^2} = \frac{(333\,000)}{(109)^2} = 28.0$$

Therefore, $g_S = 28.0\ g_E$ so the acceleration due to gravity on the sun is 28.0 times what it is on Earth. In other words, it is 28.0 times 10.0 m/s^2, or **280. m/s^2.**

Practice Exercises

Exercise 7: In *The Little Prince*, the Prince visits a small asteroid called B612. If asteroid B612 has a radius of only 20.0 m and a mass of 1.00×10^4 kg, what is the acceleration due to gravity on asteroid B612?

Answer: ____________________

Exercise 8: In Exercise 5 in the previous section, what is the Andromeda Galaxy's acceleration rate toward the Milky Way?

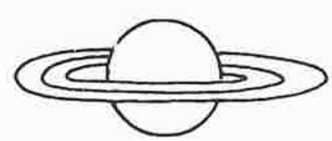
Answer: ____________________

Exercise 9: Black holes are suspected when a visible star is being noticeably pulled by an invisible partner that is more than 3 times as massive as the sun. a) If a red giant (a dying star) is gravitationally accelerated at 0.075 m/s^2 toward an object that is 9.4×10^{10} m away, how large a mass must the unseen body possess? b) How many times more massive is the object than the sun? ($M_s = 1.99 \times 10^{30}$ kg)

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 10: The planet Saturn has a mass that is 95 times Earth's mass and a radius that is 9.4 times Earth's radius. What is the acceleration due to gravity on Saturn?



Answer: ______________________

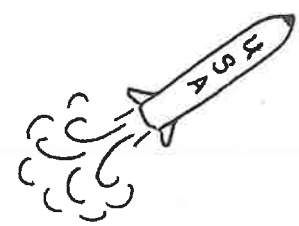
7-3 Escape Speed

Vocabulary **Escape Speed:** The minimum speed an object must possess in order to escape from the gravitational pull of a body.

In Chapter 5, you worked with gravitational potential energy and kinetic energy. When an object moves away from Earth, its gravitational potential energy increases. Since its total energy is conserved, its kinetic energy decreases. When the object is close to Earth, the gravitational force on it is a fairly constant mg. However, as you know, the gravitational force drops rapidly as you get farther from Earth. If an object moves upward from Earth with enough speed, it will never run out of kinetic energy and will escape from Earth.

The escape speed for an object leaving the surface of any celestial body of mass M and radius d is

$$v = \sqrt{\frac{2GM}{d}}$$



Notice that the mass of the escaping object does not affect the escape speed.

Solved Examples

Example 6: Earth has a mass of 5.98×10^{24} kg and a radius of 6.38×10^{6} km. What is the escape speed of a rocket launched on Earth?

Given: $M = 5.98 \times 10^{24}$ kg
$d = 6.38 \times 10^{6}$ m
$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Unknown: $v = ?$
Original equation: $v = \sqrt{\frac{2GM}{d}}$

Solve: $$v = \sqrt{\frac{2GM}{d}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{6.38 \times 10^{6}\ \text{m}}}$$

$= \mathbf{11\ 200\ m/s}$

Any rocket trying to escape Earth's gravitational pull must be going at least 11 200 m/s before engine cut-off, in order to get away.

Example 7: Compare Example 6 with the escape speed of a rocket launched from the moon. The mass of the moon is 7.35×10^{22} kg and the radius is 1.74×10^{6} m.

Given: $M = 7.35 \times 10^{22}$ kg
$d = 1.74 \times 10^{6}$ m
$G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Unknown: $v = ?$
Original equation: $v = \sqrt{\frac{2GM}{d}}$

Solve: $v = \sqrt{\frac{2GM}{d}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{1.74 \times 10^6\ \text{m}}} = \mathbf{2370\ m/s}$

Notice that you can escape from the moon by traveling much more slowly than you must travel to escape the gravitational pull of Earth. This is why launching a Lunar Module from the moon's surface was so much easier than launching an *Apollo* spacecraft from Earth.

Practice Exercise

Exercise 11: How fast would you need to travel a) to escape the gravitational pull of the sun? ($M_S = 1.99 \times 10^{30}$ kg, $d_S = 6.96 \times 10^8$ m) b) As the sun begins to die, it will become a red giant. This means that its mass will remain the same but its diameter will increase substantially (perhaps even out as far as Earth's orbit!). When the sun becomes a red giant, will its escape speed be greater than, less than, or the same as, it is now?

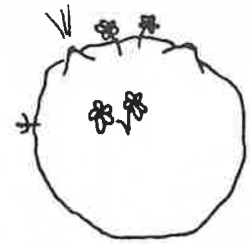
Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 12: How fast would the moon need to travel in order to escape the gravitational pull of Earth, if Earth has a mass of 5.98×10^{24} kg and the distance from Earth to the moon is 3.84×10^8 m?

Answer: ____________________

Exercise 13: What is the escape speed needed a) to escape the gravitational pull of Asteroid B612 (see Exercise 7)? b) What would happen if you jumped up on Asteroid B612?



Answer: **a.** ____________________

Answer: **b.** ______________________________

Exercise 14: Scotty finds it difficult to play catch on planet Apgar because the planet's escape speed is only 5.00 m/s, and if Scotty throws the ball too hard, it flies away. If planet Apgar has a mass of 1.56×10^{15} kg, what is its radius?

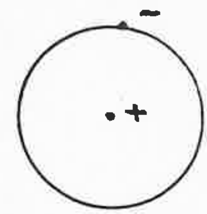
Answer: ____________________

Additional Exercises

A-1: Halley's Comet orbits the sun about every 75 years due to the gravitational force the sun provides. Compare the gravitational force between Halley's Comet and the sun when the comet is at aphelion (its greatest distance from the sun) and d is about 4.5×10^{12} m to the force at perihelion (or closest approach), where d is about 5.0×10^{10} m.

A-2: In Exercise A-1, what is the comet's acceleration a) at aphelion? b) at perihelion? ($M_S = 1.99 \times 10^{30}$ kg)

A-3: An early planetary model of the hydrogen atom consisted of a 1.67×10^{-27}-kg proton in the nucleus and a 9.11×10^{-31}-kg electron in orbit around it at a distance of 5.0×10^{-11} m. In this model, what is the gravitational force between a proton and an electron?



A-4: At what height above Earth would a 400.0-kg weather satellite have to orbit in order to experience a gravitational force half as strong as that on the surface of Earth?

A-5: It is said that people often behave in unusual ways during a full moon. a) Calculate the gravitational force that the moon would exert on a 50.0-kg student in your physics class. The moon is 3.84×10^{8} m from Earth and has a mass of 7.35×10^{22} kg. b) Does the moon attract the student with a force that is greater than, less than, or the same as the force with which the student attracts the moon?

A-6: The tiny planet Mercury has a radius of 2400 km and a mass of 3.3×10^{23} kg. a) What would be the gravitational acceleration of an astronaut standing on the surface of Mercury? b) Compare the motion of a ball dropped on the surface of Mercury to that of a ball dropped on Earth.

A-7: The acceleration due to gravity on Venus is 0.89 that of Earth. a) If the radius of Venus is 6.05×10^{6} m, what is Venus' mass? b) How does this compare to Earth's mass? c) If you were on a diet and had to "weigh in," would you rather stand on a scale on Venus or on Earth in order to appear as if you had lost the most weight?

A-8: The planet Mars has a mass that is 0.11 times Earth's mass and a radius that is 0.54 times Earth's radius. a) How much would a 60.0-kg astronaut weigh if she were to stand on the surface of Mars? b) Although Mercury is much smaller than Mars, it has almost the same gravitational acceleration. Describe how you might explain this phenomenon.

A-9: On October 26, 2000, the NEAR Shoemaker spacecraft swooped within 3 miles of the asteroid Eros, taking images and collecting data from a distance closer than any spacecraft has ever come to an asteroid. Eros has a mass of 6.69×10^{15} kg. The strange potato-like shape of Eros makes its diameter difficult to determine. If the NEAR spacecraft is orbiting a distance of 18 300 m from Eros' center of mass, what gravitational acceleration does Eros provide on NEAR?

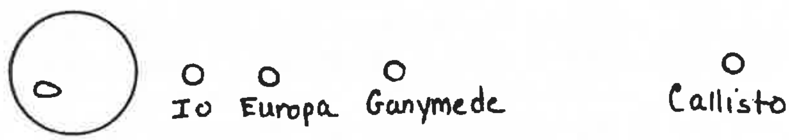
A-10: Find the NEAR spacecraft's escape speed from Eros, using the information given in A-9.

A-11: NASA has announced that a mission to Mars to return rock samples to Earth could come as early as 2011. If NASA landed a 360.-kg spacecraft on the surface of Mars a) what would be the weight of the spacecraft on the planet's surface b) what escape speed would be needed for the craft to leave the planet and head back to Earth with its rock samples. ($M_m = 6.42 \times 10^{23}$ kg, $d_M = 3.39 \times 10^{6}$ m)

Challenge Exercises for Further Study

B-1: At what distance from Earth's center must a spacecraft be in order to experience the same gravitational attraction from both Earth and the moon when directly between the two? ($M_E = 5.98 \times 10^{24}$ kg, $M_M = 7.35 \times 10^{22}$ kg $d_{E-M} = 3.84 \times 10^8$ m)

B-2: Jupiter's innermost Galilean satellite, Io, is covered with active volcanoes, which exist because of the immense gravitational tugging on the satellite by Jupiter and the other moons near Io. Io orbits 4.2×10^8 m from the center of Jupiter. The other Galilean satellites are located as follows from Jupiter's center. Europa: 6.7×10^8 m, Ganymede: 1.0×10^9 m, and Callisto: 1.9×10^9 m. If Jupiter and its satellites are lined up as shown, what gravitational force does the satellite Io experience? ($M_I = 8.9 \times 10^{22}$ kg, $M_E = 4.9 \times 10^{22}$ kg, $M_G = 1.5 \times 10^{24}$ kg, $M_C = 1.1 \times 10^{23}$ kg, $M_J = 1.9 \times 10^{27}$ kg)



B-3: Saturn's satellite, Titan, orbits the planet in a little less than 16 days. Titan orbits Saturn at an average distance of 1.216×10^9 m from the center of the planet. Use this information to find the mass of Saturn.