8 Special Relativity

8-1 Time Dilation

Relative Speed

The speed of an object depends upon what frame of reference you use to measure that speed. If a ball is thrown forward out of a stationary car at 4 m/s, the ball will continue to travel horizontally at 4 m/s until it hits the ground. However, if the car is in motion, a number of different things can happen.

For example, according to a stationary observer, if the car is moving forward at 10 m/s, the ball is also traveling 10 m/s in addition to the 4 m/s given to the ball when it is thrown. Therefore, it has a speed relative to the observer of 14 m/s.

Now, if the ball is thrown at 4 m/s in a direction opposite to the car's motion, the initial 4 m/s with which the ball is thrown is subtracted from the 10 m/s speed of the forward-moving car. The ball has a speed of 6 m/s relative to a stationary observer. It is still moving in the same direction as the car but at a reduced speed with respect to the ground.

Time Dilation

When an object (such as a spaceship) is traveling near the speed of light, the time interval between two events that occur at the same place on the moving object seems longer from the perspective of a stationary observer than it does from the perspective of the moving observer. In other words, time appears to be **dilated,** or stretched out. The stationary observer thinks that the traveler's clock has slowed down. This dilation is written as

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - (v^2/c^2)}}$$

where Δt is the time interval between two events, as measured by an observer who is in motion with respect to the events;
Δt_o is the time interval between two events, as measured by an observer who is at rest with respect to the events (also called the proper time);
v is the speed of the moving object;
c is the speed of light.

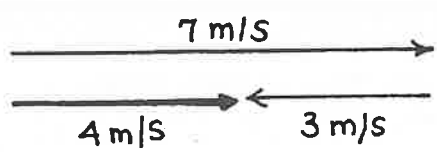
Therefore, when a spaceship is traveling close to the speed of light, its inhabitants will appear to age more slowly and, in fact, all events will occur more slowly from the perspective of an Earth-based observer.

In physics, astronomical distances are often written with the unit **light-years, (ly)**. A light-year is the distance light travels in 1 year. It is equivalent to 9.46×10^{15} m.

Solved Examples

Example 1: Farmer MacGregor is throwing bales of hay off the back of his hay wagon with a speed of 3 m/s relative to the wagon, which is pulled by a tractor moving forward with a speed of 7 m/s. With what horizontal velocity do the bales of hay hit the ground?

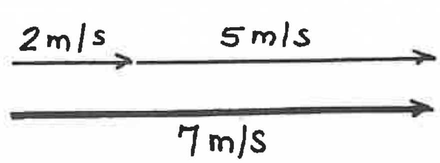
Solution: First, consider the direction of each of the velocities and treat them as vectors. Relative to the truck, the bales of hay are traveling at 3 m/s. However, relative to the ground, the speed is somewhat different, as shown.



$v = 7\text{ m/s} - 3\text{ m/s} =$ **4 m/s (in the direction of the tractor's motion)**

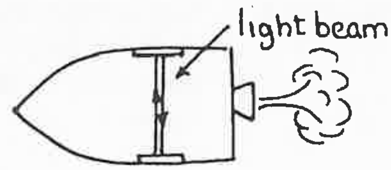
Example 2: Monty is being pulled in his wagon with a speed of 2 m/s when he tosses in front of the wagon a Frisbee whose speed is 5 m/s relative to the ground. Neglecting air resistance, how fast is the Frisbee moving when his dog, Snoopy, catches it in his mouth?

Solution: The wagon is moving at 2 m/s while the Frisbee travels an additional 5 m/s in the same direction. Therefore,



$v = 2\text{ m/s} + 5\text{ m/s} =$ **7 m/s forward,** relative to the ground.

Example 3: A light beam takes 3.0×10^{-8} s to bounce back and forth vertically between two mirrors inside a moving spaceship, according to an observer on board the spaceship. How long would the beam take according to Gerard, a stationary observer on Earth, if the spacecraft were moving directly overhead in a direction perpendicular to the line of sight with a speed of $0.60c$?



Solution: The number $0.60c$ in the exercise means that the speed of the spacecraft is 6/10 the speed of light. The speed of light is represented with the letter c.

Given: $\Delta t_o = 3.0 \times 10^{-8}$ s
$v = 0.60c$

Unknown: $\Delta t = ?$
Original equation: $\Delta t = \dfrac{\Delta t_o}{\sqrt{1-(v^2/c^2)}}$

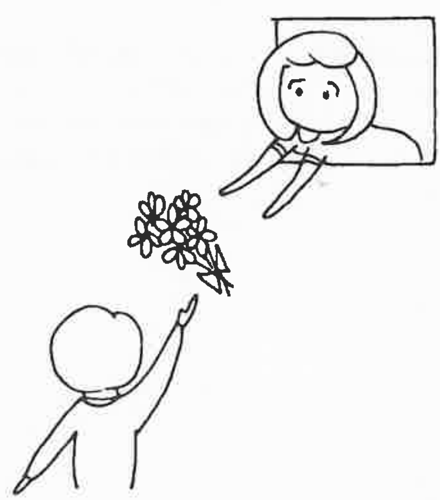
Solve: $$\Delta t = \frac{\Delta t_o}{\sqrt{1-(v^2/c^2)}} = \frac{3.0 \times 10^{-8}\text{ s}}{\sqrt{1-[(0.60c)^2/c^2]}} = \frac{3.0 \times 10^{-8}\text{ s}}{\sqrt{1-0.36}} = \frac{3.0 \times 10^{-8}\text{ s}}{\sqrt{0.64}}$$

$$= \mathbf{4.7 \times 10^{-8}\ s}$$

Therefore, if the spacecraft is traveling at $0.60c$, a time interval of 3.0×10^{-8} s according to the clocks on the spacecraft actually takes 4.7×10^{-8} s according to the clocks on Earth.

Practice Exercises

Exercise 1: Fiona is on her way home from France but she must leave her new-found love, Pierre, behind. As Fiona's train pulls out of the station at 4 m/s, Pierre tosses Fiona a bouquet of flowers with a speed of 6 m/s. According to Fiona, how fast are the flowers moving when she catches them?



Answer: ______________________

Exercise 2: Skip is bringing his boat into port with a speed of 7 m/s and as he nears the dock, he tosses a tow rope from the bow with a speed of 3 m/s to a waiting dock worker. How fast is the rope moving when it is caught by the dock worker?

Answer: ____________________

Exercise 3: Superman leaves Lois in Metropolis to rescue a malfunctioning space probe sent up from Earth. Flying at a speed of $0.70c$, Superman reaches the probe in 20. hours according to his wristwatch. How long would the trip take according to Lois's clock on Earth?

Answer: ____________________

Exercise 4: An elementary particle called a pion has a lifetime of 2.6×10^{-8} s when at rest. a) Will its lifetime be longer or shorter, as viewed from the stationary frame of reference, if it is made to travel at $0.80c$? b) What will its lifetime be according to a stationary observer?

Answer: **a.** ____________________

Answer: **b.** ____________________

Exercise 5: It is the year 3539 and, on his 30th birthday, Albert leaves for a new job opening on the planet Zil. After saying good-bye to his twin brother Henry, Albert jumps in his spacecraft and takes off for Zil traveling at a speed of $0.95c$. The total trip takes 3.0 years according to the clocks on board Albert's spaceship. How old are Henry and Albert when the three-year journey is complete?

Answer: ____________________

Exercise 6: The brightest star visible from the northern hemisphere is the star Sirius, which is 8.7 light-years from Earth in the constellation of Canis Major. It takes a spaceship 4.9 y to travel from Earth to Sirius, according to the spaceship's on-board clocks. According to Earth clocks, the trip takes 10.0 years. At what fraction of the speed of light did the spacecraft travel?

Answer: ____________________

8-2 Relativistic Length and Energy

Length Contraction

When an object is traveling close to the speed of light, the length of that object along the direction of motion appears to shrink as seen by a stationary observer. In other words, length appears to **contract.** This contraction is written as

$$L = L_o \sqrt{1 - (v^2/c^2)}$$

where L is the length (or distance) between two points as measured by an observer who is in motion with respect to the points;
L_o is the length (or distance) between two points as measured by an observer who is at rest with respect to the two points (also called the proper length);
v is the speed of the moving object;
c is the speed of light.

Therefore, when a spaceship is traveling close to the speed of light, the spaceship itself will seem shorter, from the perspective of an Earth-based observer.

Energy

Einstein proposed the idea that anything that has mass has energy. This energy is called the **rest energy, E_o**. It is measured in joules.

rest energy = (rest mass)(speed of light)^2 or $E_o = m_o c^2$

If the object is moving at a speed v, the total energy of the object is greater, as measured by a stationary observer.

$$E = mc^2 = \frac{E_o}{\sqrt{1 - (v^2/c^2)}} = \frac{m_o c^2}{\sqrt{1 - (v^2/c^2)}}$$

where E is the total energy of an object as measured by an observer who is in motion with respect to the object;
E_o is the total energy of an object as measured by an observer who is at rest with respect to the object;
m is the mass of the object as measured by an observer who is in motion with respect to the object;
m_o is the mass of the object as measured by an observer who is at rest with respect to the object;
v is the speed of the moving object;
c is the speed of light.

Solved Examples

Example 4: The star Betelgeuse in the constellation of Orion is 520 light-years away as perceived by an observer on Earth. If a space traveler journeyed to Betelgeuse at 99% the speed of light (0.99*c*), how long would this distance be according to the traveler?

Given: $L_o = 520 \text{ ly}$
$v = 0.99c$

Unknown: $L = ?$
Original equation: $L = L_o\sqrt{1 - (v^2/c^2)}$

Solve: $L = L_o\sqrt{1 - (v^2/c^2)} = (520 \text{ ly})\sqrt{1 - [(0.99c)^2/c^2]} = (520 \text{ ly})\sqrt{0.14}$
$= \mathbf{73 \text{ ly}}$

So the distance to the star would appear considerably shorter to the space traveler.

Example 5: An electron and a positron meet, each with a rest mass of 9.11×10^{-31} kg, and are converted to energy (gamma rays). How much energy is converted from the rest energy into gamma rays in the collision?

Solution: Because the mass in this exercise is actually the combination of the electron's mass and the positron's mass, add these two masses together to obtain 1.82×10^{-30} kg.

Given: $m_o = 1.82 \times 10^{-30} \text{ kg}$
$c = 3.00 \times 10^8 \text{ m/s}$

Unknown: $E = ?$
Original equation: $E = m_oc^2$

Solve: $E = m_oc^2 = (1.82 \times 10^{-30} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = \mathbf{1.64 \times 10^{-13} \text{ J}}$

Example 6: A 10 000.-kg meteor falls to Earth from space. a) What is the rest energy of the meteor? b) When it is traveling at a speed of 0.0400*c*, what is the meteor's energy according to an observer on Earth?

a. *Given:* $m_o = 10\,000. \text{ kg}$
$c = 3.00 \times 10^8 \text{ m/s}$

Unknown: $E_o = ?$
Original equation: $E_o = m_oc^2$

Solve: $E_o = m_oc^2 = (10\,000. \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 = \mathbf{9.00 \times 10^{21} \text{ J}}$

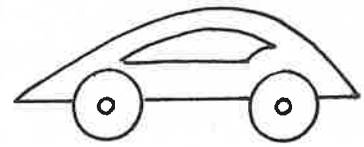
b. *Given:* $E_o = 9.00 \times 10^{21} \text{ J}$
$v = 0.0400c$

Unknown: $E = ?$
Original equation: $E = \dfrac{E_o}{\sqrt{1 - (v^2/c^2)}}$

Solve: $E = \dfrac{E_o}{\sqrt{1 - (v^2/c^2)}} = \dfrac{9.00 \times 10^{21} \text{ J}}{\sqrt{1 - [(0.0400c)^2/c^2]}} = \mathbf{9.01 \times 10^{21} \text{ J}}$

Practice Exercises

Exercise 7: The year is 2100, and a sports car company claims to have invented a new car that can travel at 0.500*c*. You take one of these cars for a test drive past your house, which is 15.0 m wide. How wide does your house appear to be when the car is up to full speed?



Answer: ______________________

Exercise 8: A stretch limo of the future is 8.0 m long but appears to be only 6.0 m long when driven at speeds near the speed of light. How fast must Linda the limo driver be going to make the limo appear 6.0 m long to an outside observer?

Answer: ______________________

Exercise 9: The starship *Enterprise* is traveling past Jupiter at a speed of 0.7500*c*. a) If Jupiter has a diameter of 142 796 km, how wide is Jupiter according to the crew of the *Enterprise*? b) What shape will Jupiter appear to have?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 10: The net result of a hydrogen fusion reaction is that four hydrogen atoms combine to form one helium atom. The mass lost when rest energy is converted into radiation energy in the reaction is 4.59×10^{-29} kg. a) How much radiation energy does this reaction produce? b) In what form can this energy be observed here on Earth?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 11: A nuclear reactor releases 9.1×10^{13} J of energy during fission. How much mass is needed to create this amount of energy?

Answer: ____________________

Exercise 12: In particle accelerators such as the CERN accelerator in Geneva, Switzerland, particles are accelerated to speeds near that of light. If a proton of rest mass 1.67×10^{-27} kg travels at a speed of $0.95c$, what is the total energy of the proton according to a stationary observer?

Answer: ____________________

Additional Exercises

A-1: Juan gets on the school bus in the morning and, as the driver starts to pull away, Juan's mother runs toward the bus with Juan's lunch bag in her hand. When the bus is traveling at a speed of 12 m/s, Mom tosses the lunch bag to Juan and he reaches out the open window to catch it. If the bag is moving 3 m/s according to Juan, with what speed did Mom throw the lunch bag?

A-2: Ming and Wong are playing a game of table tennis in the recreation car of a train. Each boy hits the ball with a speed of 20 m/s. a) If the train is traveling at 30 m/s, describe the speed of the ball as seen by an observer standing on the ground behind the railroad crossing guardrail. b) How would Ming and Wong describe the ball's speed?

A-3: The year is 2092 and the beings of the planet Quigg have captured an alien whom they are transporting home to show to the Quiggians. The trip takes 5.0 years, traveling at a speed of $0.80c$ according to the alien's on-board clock. How long will the trip take according to the inhabitants of planet Quigg?

A-4: SpaceTours, Inc. is booking passage on a ship that will travel through space at a speed of $0.70c$. The journey will last 5.0 years according to a stationary observer on Earth. How long will Margie, one of the passengers, be gone according to the on-board clock?

A-5: As the Rebel Forces fly by the *Death Star* at a speed of $0.980c$, what is the apparent diameter of the *Death Star* if its actual diameter is 7000. m?

A-6: Every time Pinocchio tells a lie, his nose grows 1.0 cm. In the past few weeks Pinocchio has told many lies and his nose is now 10.0 cm long. How fast must Pinocchio travel to make his nose appear to be 2.0 cm long to a stationary observer?

A-7: Assume that all of the radiation energy in the Big Bang was converted into the rest energy of the matter that is now the known universe. If the universe has a rest mass of 10^{51} kg, how much radiation energy will be released if the universe eventually undergoes a "Big Crunch"?

A-8: At the Bates Linear Accelerator in Middleton, Massachusetts, electrons are accelerated to near-light speeds inside a giant underground tunnel. If a 9.11×10^{-31}-kg electron is traveling at $0.89c$, what is its total energy?

Challenge Exercises for Further Study

B-1: You are riding in a station wagon on a two-lane highway at 30 m/s, and you pass a sedan going in the other direction at 25 m/s.

a) Why does the sedan appear to be moving so much faster than you are?
b) How fast does the sedan appear to be moving from your perspective?
c) How fast does your station wagon appear to be moving relative to the sedan?
d) How would your answers to (b) and (c) change if the sedan were moving in the same direction as your car?
e) How fast does each car appear to be moving to an observer standing by the side of the road?

B-2: Leon observes that his heart beats 60.0 times per minute from his own frame of reference.

a) If Leon gets into a rocket on his 15th birthday and flies away from Earth fast enough so that his heartbeat appears to occur half as frequently as observed from Earth, how fast is the rocket traveling?
b) How old is Leon if he returns to Earth after Earth-based clocks said that he had been gone for 12.0 years?