11 Simple Harmonic Motion

11-1 Springs

Vocabulary **Period:** The time it takes for a vibrating object to repeat its motion.

Vocabulary **Frequency:** The number of vibrations made per unit time.

Period and frequency are the reciprocals of each other. In other words,

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Since period is a measure of time, its SI unit is the **second,** while the unit for frequency is the reciprocal of this, or **1/second.** Another way of writing 1/s is with the unit **hertz (Hz).**

You may recognize this as being similar to the explanation of period and frequency in Chapter 6 on circular motion.

Hooke's Law

Whenever a spring is stretched from its equilibrium position and released, it will move back and forth on either side of the equilibrium position. The force that pulls it back and attempts to restore the spring to equilibrium is called the **restoring force.** Its magnitude can be written as

restoring force = (force constant)(displacement from equilibrium)
or $F = kx$

This relationship is known as **Hooke's law.** The force constant is a measure of the stiffness of the spring. The SI unit for the force constant is the **newton per meter (N/m).**

Keep in mind that this is the force required to restore the spring back to its original position. The force that acts to move the spring *away* from the equilibrium position is equal in magnitude to the restoring force, but opposite in direction.

Simple harmonic motion is motion that occurs when the restoring force acting on an object is proportional to the object's displacement from its rest position. Objects at the end of springs move in simple harmonic motion when they are displaced from their rest position and bounce up and down on the spring, or oscillate.

Period of a Mass on a Spring in Simple Harmonic Motion

The only two things that affect the period of an object hanging on a bouncing spring are the object's mass and the force constant of the spring on which the object is oscillating.

$$\textbf{Period} = 2\pi\sqrt{\frac{\textbf{mass}}{\textbf{force constant}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

To prove that this equation does indeed give the period in seconds, simplify the units for $\sqrt{m/k}$ by writing

$$\sqrt{\frac{\text{kg}}{\text{N/m}}} = \sqrt{\frac{\text{kg}}{\frac{\text{kg}\cdot\text{m/s}^2}{\text{m}}}} = \sqrt{\text{s}^2} = \text{s}$$

Solved Examples

Example 1: A hummingbird beats its wings up and down with a frequency of 80.0 Hz. What is the period of the hummingbird's flaps?

Given: $f = 80.0$ Hz

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{80.0\text{ Hz}} = \mathbf{0.0125\ s}$

Example 2: In anticipation of her first game, Alesia pulls back the handle of a pinball machine a distance of 5.0 cm. The force constant of the spring in the handle is 200 N/m. How much force must Alesia exert?

Solution: First, convert cm to m. 5.0 cm = 0.050 m

Given: $k = 200$ N/m
$x = 0.050$ m

Unknown: $F = ?$
Original equation: $F = kx$

Solve: $F = kx = (200\text{ N/m})(0.050\text{ m}) = \mathbf{10\ N}$

Example 3: As Bianca stands on a bathroom scale, whose force constant is 220 N/m, the needle on the scale vibrates from side to side. a) If Bianca has a mass of 180 kg, what is the period of vibration of the needle as it comes to rest? b) If Bianca goes on a diet, how will this change the period of vibration?

a. *Given:* $m = 180\text{ kg}$
$k = 220\text{ N/m}$

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\frac{m}{k}}$

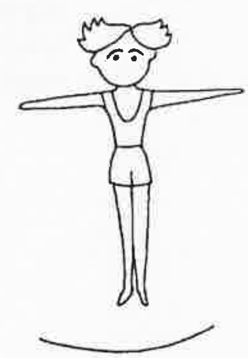
Solve: $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{180\text{ kg}}{220\text{ N/m}}} = \mathbf{5.7\ s}$

In other words, this is the amount of time for one complete oscillation.

b. Because the mass will be smaller, the period of vibration will be smaller. In other words, it will take less time for the needle on the scale to bounce from side to side as it comes to rest.

Practice Exercises

Exercise 1: Terry jumps up and down on a trampoline with a frequency of 1.5 Hz. What is the period of Terry's jumping?



Answer: ______________________

Exercise 2: Gary Stewart of Reading, Ohio set a pogo stick record in 1990 by jumping 177 737 times. a) If the pogo stick he used had a force constant of 6000. N/m and was compressed 0.12 m on each jump, what force must Gary have exerted on the pogo stick upon each jump? b) What force would be exerted back up on Gary each time he went up?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 3: At the post office, Cliff, a postal worker, places a 0.60-kg package on a scale, compressing the scale by 0.03 m. a) What is the force constant of the spring in the postal scale? b) What happens to the force constant if Cliff weighs a heavier package?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 4: A jack-in-the-box lid will pop open when a crank is turned on the outside of the box. If Jack pushes against the inside of the box with a force of 3.00 N when the lid is closed, and the spring is compressed 10.0 cm from equilibrium, what is the force constant of the spring?

Answer: ______________________

Exercise 5: Sam, a butcher, puts 3.0 kg of chopped beef on the 1.0-kg pan of his scale, which has a spring whose force constant is 400. N/m. What is the period of vibration of the pan as it comes to rest? b) If Sam adds more beef to the scale, what will this do to the period of vibration?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 6: A toy bobs up and down over Campbell's crib with a period of 1.0 s. The toy hangs from the end of a spring whose force constant is 20.0 N/m. What is the mass of the toy?

Answer: ____________________

11-2 Pendulums

The period of a pendulum depends only upon the pendulum's length (if the angle of swing is not too large). A long pendulum has a longer period than a short pendulum. The relationship between period and length can be shown with the following equation.

$$\textbf{Period} = 2\pi\sqrt{\frac{\textbf{length}}{\textbf{acceleration due to gravity}}} \quad \text{or} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

It should be noted that this equation works only for a pendulum whose mass is considerably larger than the mass of the string from which it swings. To simplify calculations, in the following exercises you will be working with pendulums swinging from strings of negligible mass.

Solved Examples

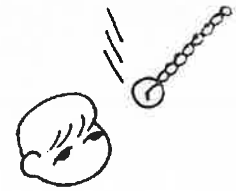
Example 4: A tall, thin tree sways back and forth in the breeze with a frequency of 2 Hz. What is the period of the tree?

Given: $f = 2\text{ Hz}$

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{2\text{ Hz}} = \mathbf{0.5\ s}$

Example 5: World-reknowned hypnotist Paulbar the Great swings his watch from a 20.0-cm chain in front of a subject's eyes. What is the period of swing of the watch?



Solution: First, convert cm to m. 20.0 cm = 0.20 m

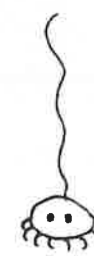
Given: $L = 0.20\ \text{m}$
$g = 10.0\ \text{m/s}^2$

Unknown: $T = ?$
Original equation: $T = 2\pi\sqrt{\frac{L}{g}}$

Solve: $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{0.20\ \text{m}}{10.0\ \text{m/s}^2}} = \mathbf{0.89\ s}$

Therefore, it takes 0.89 s for the watch to swing in one direction and back again, through one full cycle.

Example 6: A spider swings in the breeze from a silk thread with a period of 0.6 s. How long is the spider's strand of silk?



Solution: The answer is determined using the pendulum equation, but now it must be set up in terms of the unknown, L. First, square all of the terms to get rid of the radical. The equation becomes

$T^2 = 4\pi^2 \frac{L}{g}$. Then rearrange the equation as shown.

Given: $T = 0.60\ \text{s}$
$g = 10.0\ \text{m/s}^2$

Unknown: $L = ?$
Original equation: $T = 2\pi\sqrt{\frac{L}{g}}$

Solve: $L = \frac{gT^2}{4\pi^2} = \frac{(10.0\ \text{m/s}^2)(0.6\ \text{s})^2}{4\pi^2} = \mathbf{0.09\ m}$

Practice Exercises

Exercise 7: A metronome is a device used by many musicians to get the desired rhythm for a musical piece. If a metronome is clicking back and forth with a frequency of 0.5 Hz, what is the period of the metronome?

Answer: ____________________

Exercise 8: Many amusement parks feature a ride in which a giant ship swings back and forth. If the period of the ship is 8.00 s, what is the frequency of the swinging ship?

Answer: ______________________

Exercise 9: Tegan, a trapeze artist, swings from a 2.5-m-long trapeze, high above the three-ring circus. a) What is Tegan's period of swing? b) Would Tegan's period of swing change if she were more massive? If so, how?

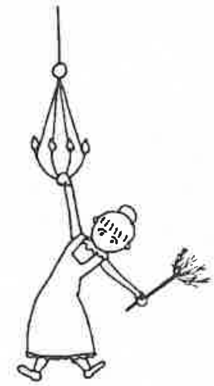
Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 10: Danielle is pushing her twin Daniel on a swing that hangs from a tree branch by 2.0-m-long ropes. With what frequency will Danielle have to push Daniel as he swings?

Answer: ______________________

Exercise 11: Marla, a maid, is standing on the Vanderbilt's dining room table dusting the chandelier. While Marla is reaching up, she slips and grabs hold of the chandelier to catch her balance. When she lets go, the chandelier begins to swing with a period of 1.6 s. How long is the cable connecting the chandelier to the ceiling?



Answer: ______________________

Exercise 12: You have been commissioned by NASA to travel to Jupiter's innermost Galilean satellite, Io, to learn more about this volcanic moon. As you board the spacecraft, you are handed a rock tied to a 10.0-cm string, and a stopwatch, and are asked to derive an experiment that would allow you to determine the acceleration due to gravity on Io. You must use both pieces of equipment and nothing more. a) Describe how you would calculate Io's gravitational acceleration. b) If the pendulum swings with a period of 1.48 s, what is the gravitational acceleration on Io?

Answer: **a.** ______________________

Answer: **b.** ______________________

Additional Exercises

A-1: Mr. Knote, a piano tuner, taps his 440-Hz tuning fork with a mallet. What is the period of the vibrating tuning fork?

A-2: Denny jumps up and down on his bed, taking 0.75 s for each jump. What is the frequency of Denny's jumping?

A-3: Inside most ball-point pens is a small spring that compresses as the pen is pressed against the paper. If a force of 0.1 N compresses the pen's spring a distance of 0.005 m, what is the force constant of the tiny spring?

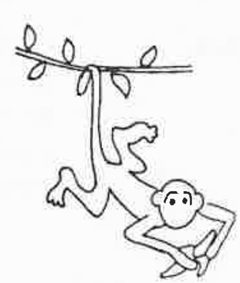
A-4: Maureen is trying to predict the period of a mass hung on a spring. She has a spring of negligible mass and four weights to hang on the end. Maureen collects the following data as she observes the stretch of the spring:

force (N)	displacement (m)
2.5	0.050
5.0	0.102
7.5	0.149
10.0	0.199

a) Plot a graph of force (on the y-axis) vs. displacement (on the x-axis). b) Find the slope of the graph. What does this slope represent? c) Use the information you have obtained to find the period of the spring when a 3.0 kg mass is hung on the end.

A-5: Kim drives her empty dump truck over a berm (also called a speed bump) at the construction site. The truck has a mass of 3000. kg and the force constant for one of the truck's springs is 100 000. N/m. (Remember, the truck has 4 wheels.) a) What is the resulting period of the bouncing truck as it goes over the bump? b) If Kim leaves the construction site with a load of dirt in her truck, what will this do to the period of her dump truck as it crosses the berm?

A-6: A monkey swings from a jungle vine by his 0.30-m-long tail. a) What is the period of swing of the monkey? b) With what frequency does the monkey swing?



A-7: A wrecking ball used to demolish buildings swings from a 10.0-m-long cable. What is the period of the wrecking ball as it swings?

A-8: A crow attempts to land on a small bird feeder, causing it to swing back and forth with a frequency of 0.350 Hz. How long is the wire from which the feeder hangs?

A-9: The acceleration due to gravity on the moon is 1/6 that on Earth. a) If you wanted a pendulum clock to tell time on the moon the same as it does on Earth (i.e., have the same period), would you need to lengthen or shorten the pendulum? b) If the pendulum was originally 24.0 cm long on Earth, how long should it be on the moon?

Challenge Exercises for Further Study

B-1: Ezra, a 60.0-kg high school student, is sleeping on his waterbed when his 2.0-kg cat, Muffin, jumps onto his back, causing Ezra to sink 2.0 cm deeper into the waterbed. a) If Muffin then jumps off Ezra from this new equilibrium position, what will be the period of Ezra's bobbing motion on the waterbed? b) Will this period slow down, speed up, or remain the same as the amplitude of the bounces gets smaller and smaller? Explain your answer.

B-2: Andy (mass 80.0 kg), Randy (mass 60.0 kg), and twins Candy and Mandy (each with a mass of 70.0 kg) climb into a 1000.-kg car, causing each of the four springs to compress 4.00 cm. Find the period of vibration of the car as it comes to rest after the four get in.

B-3: Tanja talks long distance with her boyfriend every night from her dormitory pay phone, and her phone bills are getting rather high. She has decided that she must limit each of her calls to 10 minutes. Since Tanja doesn't have a watch, she devises a unique way to time her calls. Tanja notices that the pay phones each have a cord that is 0.800 m long. Therefore, as she talks on one phone, she can swing the receiver of the adjacent phone to time her call. How many complete swings will the nearby phone receiver make before Tanja must hang up?

B-4: On a 0°C-winter day, a 10.000-m-long brass Foucault pendulum hanging in the covered entrance to the science museum swings back and forth with the rotation of Earth. The outdoor temperature variations range from 0°C in the winter to 30.0°C in the summer. How does the period of the pendulum change throughout the year? ($\alpha_{brass} = 19 \times 10^{-6}\,°C^{-1}$)

B-5: Gillian buys a pendulum clock at a discount store and discovers when she gets it home that it loses 6.00 minutes each day. a) Should she lengthen or shorten the pendulum in order for it to keep accurate time? b) If the pendulum has a period of 2.00 s, by how much must the length be changed so that the clock keeps accurate time?

12 Waves and Sound

12-1 Wave Motion

Vocabulary **Wave:** A disturbance in a medium.

In this chapter you will be working with waves that are periodic or that repeat in a regular, rhythmic pattern.

$$\textbf{wave speed} = (\textbf{wavelength})(\textbf{frequency}) \quad \text{or} \quad v = \lambda f$$

The SI unit for wave speed is the **meter per second (m/s).** The speed of sound in air increases with air temperature. For the following exercises, the speed of sound will be written as 340.0 m/s. All electromagnetic radiation including radio waves and light waves travel at the speed of light, 3.00×10^8 m/s.

The wavelength of a wave is the distance from one point on a wave to the next identical point on the same wave, for example, from crest to crest, trough to trough, or condensation to condensation. The symbol for wavelength is the Greek letter "lambda," λ.

The SI unit for wavelength is the **meter (m),** which is the same unit used for length in earlier chapters.

The SI unit for frequency is the **hertz (Hz).** When talking about the broadcast frequency of a radio station, frequencies of FM radio stations are given in megahertz, or MHz, and frequencies of AM radio stations are given in kilohertz, or kHz.

$$1 \text{ MHz} = 1 \times 10^6 \text{ Hz} \quad \text{and} \quad 1 \text{ kHz} = 1 \times 10^3 \text{ Hz}$$

Solved Examples

Example 1: Radio station WKLB in Boston broadcasts at a frequency of 99.5 MHz. What is the wavelength of the radio waves emitted by WKLB?

Given: $v = 3.00 \times 10^8$ m/s
$f = 99.5 \times 10^6$ Hz

Unknown: $\lambda = ?$
Original equation: $v = \lambda f$

Solve: $\lambda = \frac{v}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{99.5 \times 10^6 \text{ Hz}} = \mathbf{3.02\ m}$

Therefore, the distance from one point on the wave to the next identical point on the same wave is 3.02 m.

Example 2: In California, Clay is surfing on a wave that propels him toward the beach with a speed of 5.0 m/s. The wave crests are each 20. m apart. a) What is the frequency of the water wave? b) What is the period?

a. *Given:* $v = 5.0$ m/s
$\lambda = 20.$ m

Unknown: $f = ?$
Original equation: $v = \lambda f$

Solve: $f = \frac{v}{\lambda} = \frac{5.0\ m/s}{20.\ m} = \mathbf{0.25\ Hz}$

b. *Given:* $f = 0.25$ Hz

Unknown: $T = ?$
Original equation: $T = \frac{1}{f}$

Solve: $T = \frac{1}{f} = \frac{1}{0.25 \text{ Hz}} = \mathbf{4.0\ s}$

One crest comes along every 4.0 s.

Practice Exercises

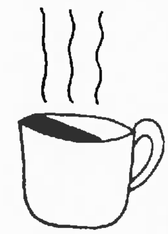
Exercise 1: Harriet is told by her doctor that her heart rate is 70.0 beats per minute. If Harriet's average blood flow in the aorta during systole is 1.5×10^{-2} m/s, what is the wavelength of the waves of blood in Harriet's aorta, created by her beating heart?

Answer: ____________________

Exercise 2: Dogs are able to hear much higher frequencies than humans are capable of detecting. For this reason, dog whistles that are inaudible to the human ear can be heard easily by a dog. If a dog whistle has a frequency of 3.0×10^4 Hz, what is the wavelength of the sound emitted?

Answer: ____________________

Exercise 3: While flying to Tucson, Connie's plane experiences turbulence that causes the coffee in her cup to oscillate back and forth 4 times each second. If the waves of coffee have a wavelength of 0.1 m, what is the speed of a wave moving through the coffee?



Answer: ____________________

Exercise 4: At a country music festival in New Hampshire, the Oak Ridge Boys are playing at the end of a crowded 184-m field when Ronny Fairchild hits a note on the keyboard that has a frequency of 400. Hz. a) How many full wavelengths are there between the stage and the last row of the crowd? b) How much delay is there between the time a note is played and the time it is heard in the last row?

Answer: **a.** ____________________

Answer: **b.** ____________________

12-2 Doppler Effect

Vocabulary

Doppler Effect: A change in the apparent frequency of sound due to the motion of the source of the receiver.

You probably associate the Doppler effect with the change in pitch (frequency) of a loud car or siren just as it passes you. The pitch suddenly drops just as the object moves by. Light can also be Doppler shifted but the Doppler shift of light will not be discussed in this chapter.

The equation that describes this effect can be used whether the source is approaching or receding from the observer. It also works if either the source or observer is at rest, or if there is a chase situation in which both are moving in the same direction.

$$\text{perceived frequency} = \text{actual frequency}\,\frac{(\text{speed of sound} + \text{speed of observer})}{(\text{speed of sound} - \text{speed of source})}$$

or $$f = f_o \frac{(v + v_o)}{(v - v_s)}$$

Here, f_o refers to the actual frequency being emitted by an object, while f is the frequency heard by the observer as the source approaches or recedes. If a source approaches, the perceived frequency will be higher than the actual frequency. If a source recedes, the perceived frequency is lower than the actual frequency.

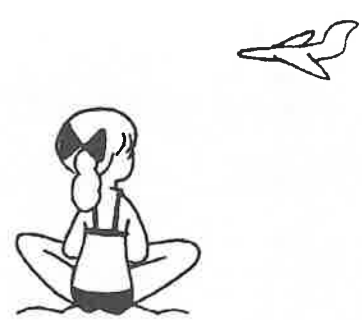
In order for this equation to work properly, there is a standard convention to which you must adhere whenever solving Doppler exercises.

v_o is (+) if the observer moves toward the source.
v_o is (−) if the observer moves away from the source.
v_s is (+) if the source moves toward the observer.
v_s is (−) if the source moves away from the observer.

Remember, it is not necessary to always have both the observer and the source in motion. Often one will be moving and the other will be at rest.

Solved Examples

Example 3: Sitting on the beach at Coney Island one afternoon, Sunny finds herself beneath the flight path of the airplanes leaving Kennedy Airport. What frequency will Sunny hear as a jet, whose engines emit sound at a frequency of 1000. Hz, flies toward her at a speed of 100.0 m/s?



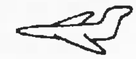
Solution: First draw a diagram of the situation. Notice in the calculation below that Sunny is sitting at rest and the plane is approaching. Therefore, the source is moving toward the observer. The observer remains stationary.

Given: $f_o = 1000.\text{ Hz}$
$v_o = 0\text{ m/s}$
$v = 340.0\text{ m/s}$
$v_s = 100.0\text{ m/s}$

Unknown: $f = ?$
Original equation: $f = f_o \frac{(v + v_o)}{(v - v_s)}$

Solve: $f = f_o \frac{(v + v_o)}{(v - v_s)} = 1000.\text{ Hz}\,\frac{(340.0\text{ m/s} + 0\text{ m/s})}{(340.0\text{ m/s} - 100.0\text{ m/s})} = \mathbf{1417\ Hz}$

Example 4: In the previous example, what frequency will Sunny observe as the jet travels away from her at the same speed?



Solution: Again, draw a diagram of the situation. This time, the source is moving away from the observer, so the value for v_s must be negative.

Given: $f_o = 1000.\ \text{Hz}$
$v_o = 0\ \text{m/s}$
$v = 340.0\ \text{m/s}$
$v_s = -100.0\ \text{m/s}$

Unknown: $f = ?$
Original equation: $f = f_o \dfrac{v + v_o}{v - v_s}$

Solve: $f = f_o \dfrac{(v + v_o)}{(v - v_s)} = 1000.\ \text{Hz} \dfrac{(340.0\ \text{m/s} + 0\ \text{m/s})}{(340.0\ \text{m/s} - [-100.0\ \text{m/s}])} = \mathbf{772.7\ Hz}$

Example 5: A sparrow chases a crow with a speed of 4.0 m/s, while chirping at a frequency of 850.0 Hz. What frequency of sound does the crow hear as he flies away from the sparrow with a speed of 3.0 m/s?

Given: $f_o = 850.0\ \text{Hz}$
$v_o = -3.0\ \text{m/s}$
$v = 340.0\ \text{m/s}$
$v_s = 4.0\ \text{m/s}$

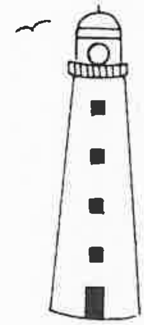
Unknown: $f = ?$
Original equation: $f = f_o \dfrac{(v + v_o)}{(v - v_s)}$

Solve: $f = f_o \dfrac{(v + v_o)}{(v - v_s)} = 850.0\ \text{Hz} \dfrac{(340.0\ \text{m/s} + [-3.0\ \text{m/s}])}{(340.0\ \text{m/s} - 4.0\ \text{m/s})} = \mathbf{852.5\ Hz}$

Therefore, since the sparrow is approaching the crow, the crow hears a frequency that is higher than the original.

Practice Exercises

Example 5: One foggy morning, Kenny is driving his speed boat toward the Brant Point lighthouse at a speed of 15.0 m/s as the fog horn blows with a frequency of 180.0 Hz. What frequency does Kenny hear as he moves?



Answer: ____________________

Example 6: Dad is driving the family station wagon to Grandma's house when he gets tired and pulls over in a roadside rest stop to take a nap. Junior, who is sitting in the back seat, watches the trucks go by on the highway and notices that they make a different sound when they are coming toward him than they do when they are moving away. a) If a truck with a frequency of 85.0 Hz is traveling toward Junior with a speed of 27.0 m/s, what frequency does Junior hear as the truck approaches? b) After the truck passes, what frequency does Junior hear as the truck moves away?

Answer: **a.** ________________

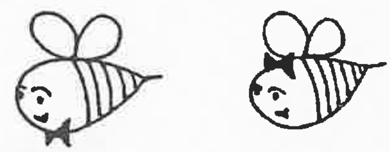
Answer: **b.** ________________

Exercise 7: One way to tell if a mosquito is about to sting is to listen for the Doppler shift as the mosquito is flying. The buzzing sound of a mosquito's wings is emitted at a frequency of 1050 Hz. a) If you hear a frequency of 1034 Hz, does this mean that the mosquito is coming in for a landing or that it has just bitten you and is flying away? b) At what velocity is the mosquito flying?

Answer: **a.** ________________

Answer: **b.** ________________

Exercise 8: Barney, a bumblebee flying at 6.00 m/s, is being chased by Betsy, a bumblebee who is flying at 4.00 m/s. Barney's wings beat with a frequency of 90.0 Hz. What frequency does Betsy hear as she flies after Barney?



Answer: ____________________

Exercise 9: Mrs. Gonzalez is about to give birth and Mr. Gonzalez is rushing her to the hospital at a speed of 30.0 m/s. Witnessing the speeding car, Officer O'Malley jumps in his police car and turns on the siren, whose frequency is 800. Hz. If the officer chases after the Gonzalez' car with a speed of 35.0 m/s, what frequency do the Gonzalezes hear as the officer approaches?

Answer: ____________________

12-3 Standing Waves

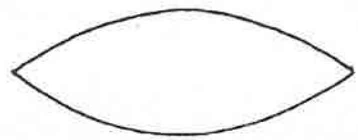
Waves in Strings

When a string is plucked, a wave will reflect back and forth from one end of the string to the other, creating **nodes** (points of minimum movement) and **antinodes** (points of maximum movement). This is called a **standing wave** because it appears to stand still.

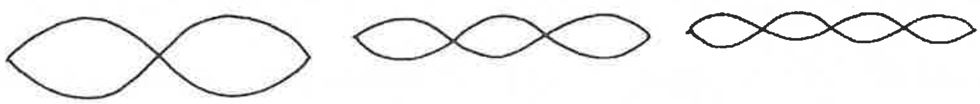
The frequency with which a string vibrates depends upon the number of antinodes, the wave speed, and the length of the string, as shown in the following relationship.

$$\textbf{frequency} = \frac{\textbf{(number of antinodes)(wave speed)}}{\textbf{2(length)}} \quad \text{or} \quad f = \frac{nv}{2L}$$

If $n = 1$, as shown in the diagram, the frequency is called the **fundamental frequency.** It is the lowest frequency of a vibrating string that is fixed at both ends. Multiples of the fundamental frequency are called **overtones.**



First overtone | **Second overtone** | **Third overtone**



$$f = \frac{2v}{2L} = \frac{v}{L} \qquad f = \frac{3v}{2L} \qquad f = \frac{4v}{2L} = \frac{2v}{L}$$

and so on.

Waves in Pipes

Waves in pipes that are open at both ends behave much like waves in strings. It is important to remember that antinodes always form at open ends of a pipe while nodes form at closed ends. If a pipe is open at both ends, the possible frequencies are

$$f = \frac{nv}{2L} \quad \text{(where } n = 1, 2, 3 \ldots \text{ for other overtones)}$$

In a pipe that is closed at one end, the possible frequencies are

$$f = \frac{nv}{4L} \quad \text{(where } n = 1, 3, 5, 7 \ldots \text{ for other overtones)}$$

Beats

If two different frequencies sound simultaneously, the wavelengths will differ, and the crests and troughs of each wave will overlap in a way that causes variations in loudness. There will be moments of reinforcement and moments of cancellation as the wave patterns interact. The resulting sound is a series of **beats,** which occur when the wave sum reaches its greatest amplitude.

The beat frequency can be found by taking the absolute value of the difference between the two frequencies of the interacting waves.

$$f_{\text{beat}} = |f_1 - f_2|$$

Solved Examples

Example 6: An orchestra tunes up for the big concert by playing an A, which resounds with a fundamental frequency of 440. Hz. Find the first and second overtones of this note.

The first overtone is 2 times the fundamental frequency:

$$f_2 = 2f_0 \quad \text{so} \quad f_2 = 2(440.\text{ Hz}) = \mathbf{880.\ Hz}$$

The second overtone is 3 times the fundamental frequency:

$$f_3 = 3f_0 \quad \text{so} \quad f_3 = 3(440.\text{ Hz}) = \mathbf{1320\ Hz}$$

Example 7: Zeke plucks a C on his guitar string, which vibrates with a fundamental frequency of 261 Hz. The wave travels down the string with a speed of 400. m/s. What is the length of the guitar string? b) Would Zeke need longer or shorter strings to play the fundamental frequency for higher notes?

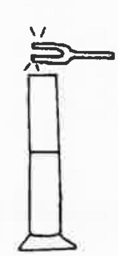
a. *Given:* $n = 1$
$v = 400.\text{ m/s}$
$f = 261\text{ Hz}$

Unknown: $L = ?$
Original equation: $f = \frac{nv}{2L}$

Solve: $L = \frac{nv}{2f} = \frac{(1)(400.\text{ m/s})}{2(261\text{ Hz})} = \mathbf{0.766\ m}$

b. If the wave speed remains the same for each string, as f gets larger, L gets smaller. Therefore, the higher the note, the shorter the string required to hear the fundamental frequency.

Example 8: In his physics lab, Sanjiv finds that he can take a long glass tube and fill it with water, using the air space at the top to simulate a pipe closed at one end. If Sanjiv holds a tuning fork, which vibrates with a fundamental frequency of 440 Hz, over the mouth of the pipe, how long is the air column if it vibrates at the same frequency?



Given: $f = 440\text{ Hz}$
$v = 340.0\text{ m/s}$
$n = 1$

Unknown: $L = ?$
Original equation: $f = \frac{nv}{4L}$

Solve: $L = \frac{nv}{4f} = \frac{(1)(340.0\text{ m/s})}{4(440\text{ Hz})} = \mathbf{0.19\ m}$

Practice Exercises

Exercise 10: Melody puts a fret on her guitar string, causing it to vibrate with a fundamental frequency of 250 Hz as a wave travels through at 350 m/s. a) How long is the guitar string from the lower fixed end to the fret? b) How far and in which direction must the fret be moved in order to produce a fundamental frequency that is twice as high (i.e., one octave higher)?

Answer: **a.** ______________________

Answer: **b.** ______________________

Exercise 11: The fundamental frequency of a bass violin string is 1045 Hz and occurs when the string is 0.900 m long. How far from the lower fixed end of the bass violin should you place your fingers to allow the string to vibrate at a fundamental frequency 3 times as great?

Answer: ______________________

Exercise 12: Aaron blows across the opening of a partially filled 20.0-cm-high soft drink bottle and finds that the air vibrates with a fundamental frequency of 472 Hz. How high is the liquid in the bottle?

Answer: ______________________

Exercise 13: A red-headed piano tuner from Chicago is tuning the Bentz' piano when he discovers that the G above middle C is vibrating with a higher frequency than his G tuning fork, which vibrates at 392.0 Hz. He plays the piano key and tuning fork at the same time and hears a beat frequency of 2.0 Hz. What is the frequency of the G on the Bentz' piano?

Answer: ____________________

Additional Exercises

A-1: Find the wavelength of the ultrasonic wave emitted by a bat if it has a frequency of 4.0×10^4 Hz.

A-2: Radio station KSON in San Diego broadcasts at both 1240 kHz (AM) and 97.3 MHz (FM). a) Which of these signals, AM or FM, has the longer wavelength? b) How long is each?

A-3: What is the wavelength of a B note (frequency 494 Hz) played a) by a flute? b) If the flute and a sax play the same note, which of the following will be different: quality, pitch, or loudness?

A-4: As an anchor is being hoisted out of the water, it hits the hull of the ship, causing the anchor to vibrate with a frequency of 150. Hz. If the speed of sound in sea water is 1520 m/s, how many wavelengths of sound will fit between the boat and the ocean bottom 395 m below?

A-5: A popular pastime at sporting events is "the wave," a phenomenon where individuals in the crowd stand up and sit down in sequence, causing a giant ripple of people. If a continuous "wave" passes through a stadium of people with a speed of 20 m/s and a frequency of 0.5 Hz, what is the distance from "crest" to "crest" (in other words, the wavelength of the wave)?

A-6: From his bedroom, Garth can hear the distant sound of a train horn as the train travels through the mountains on its way from Chattanooga to Nashville. The horn has a frequency of 800.0 Hz as the train rolls along at 20.00 m/s. What frequency does Garth hear as the train travels away?

A-7: Erin is late to physics class and is coming down the hall as the bells are ringing. There are two bells in the hall, one at the far end, and one in front of the classroom she is approaching. Each rings with a frequency of 500.0 Hz. As Erin comes down the hallway with a speed of 1.000 m/s toward the classroom a) what frequency does she hear for each bell? b) What beat frequency does she hear?

A-8: Karen flies a motorized toy airplane with a frequency of 200. Hz in a circle at a speed of 18.0 m/s. Caroline stands nearby and hears a Doppler shift as the plane approaches and recedes from her. What are the a) highest and b) lowest frequencies Caroline hears?

A-9: Sonar detectors work by bouncing high-frequency sound waves of about 0.100 MHz off oncoming ships and detecting the frequency of the return signal. If a sonar detector receives a return signal of 0.101 MHz from a sub, how fast is the sub going? (Hint: Sonar travels in sea water at 1520 m/s).

A-10: A fly traveling at 3.000 m/s is pursued by a bat traveling at 6.000 m/s who emits sound at an ultrasonic frequency of 50 000. Hz. If the fly could detect such a high frequency emission, what frequency would the fly hear as it is being pursued?

A-11: Lars is jogging beside the railroad tracks at a speed of 2.00 m/s when he hears a train whistle behind him at a frequency of 2115 Hz. If the actual frequency of the train whistle is 2000. Hz, how fast is the train moving?

A-12: Walter is a bass and can hit a low E that has a frequency of 82.4 Hz. Millie is a soprano and can sing as high as the third overtone of this note. What is the highest frequency that Millie can sing?

A-13: Joyce, the church organist, is practicing on the organ and she finds that the first two overtones for the 370-Hz pipe are 1110 Hz and 1850 Hz. Is the organ pipe closed at one end or open at both ends?

A-14: A train passes through a tunnel that is 550 m long. What is the fundamental frequency of vibrating air in the tunnel?

A-15: Harvey, a harpist, plucks a 0.600-m-long string on his harp. The string has a first overtone of 1046.6 Hz. How fast does the vibration travel through the string?

A-16: Reed arrives late to practice and finds that the orchestra has already tuned up and begun to play. As one oboist hits a D with a frequency of 293.7 Hz, Reed plays a note with a frequency of 291.2 Hz. What beat frequency is heard as the two instruments are playing side by side?

Challenge Exercises for Further Study

B-1: As a train approaches a ringing crossing gate, Stacey, a passenger on the train, hears a frequency of 440 Hz from the bell. As the train recedes, she hears a frequency of 410 Hz. How fast is the train traveling?

B-2: Richard stands on the flatbed car of a moving train playing an A on his horn. The note has a fundamental frequency of 220 Hz. Calculate whether or not the train could move fast enough for a stationary observer on the ground to hear the first overtone of the horn as the train passes.