18 Modern Physics

18-1 The Atom and the Quantum

Photon Energy

Vocabulary **Quantum:** A packet of energy that exhibits both particle and wave properties.

A quantum of light energy is called a **photon**. The photon's energy is directly proportional to the frequency of its lights. This can be written as

$$\textbf{energy} = \textbf{(Planck's constant)(frequency)} \qquad \text{or} \qquad E = hf$$

where **Planck's constant, *h*,** is equal to **6.63×10^{-34}** J·s.

Recall from Chapter 13 that $c = \lambda f$. So, the frequency of light can be written as $f = \frac{c}{\lambda}$. Therefore, the energy of a photon is

$$\textbf{energy} = \frac{\textbf{(Planck's constant)(speed of light)}}{\textbf{wavelength}} \qquad \text{or} \qquad E = \frac{hc}{\lambda}$$

The common unit for the wavelength of light is the **nanometer (nm),** which equals 10^{-9} m.

It is important to note that because the energy of a photon is so small, scientists rarely use the unit "joule" when describing this energy. Instead, a smaller unit, the **electron volt (eV)** is more commonly used in equations involving photon energy. Note that the electron volt is a unit of energy and not a unit of potential difference.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

De Broglie Waves

Vocabulary **De Broglie Wavelength:** The effective wavelength of a moving particle.

Recall that photons of light exhibit both particle and wave properties. According to de Broglie, if a moving particle of matter has a high momentum, it exhibits wave properties and has a measurable wavelength.

The de Broglie wavelength of any particle can be found from the equation

$$\textbf{wavelength} = \frac{\textbf{Planck's constant}}{\textbf{(mass)(velocity)}} \quad \text{or} \quad \lambda = \frac{h}{mv}$$

Solved Examples

Example 1: Glenn is a DJ at his high school radio station WPAA, which broadcasts at a frequency of 91.7 MHz. When the station is on the air, how much energy does each emitted photon possess a) in joules? b) in electron volts?

Solution: The term MHz means megahertz or 10^6 Hz. Therefore, 91.7 MHz means 91.7×10^6 Hz.

a. *Given:* $f = 91.7 \times 10^6$ Hz
$h = 6.63 \times 10^{-34}$ J·s

Unknown: $E = ?$
Original equation: $E = hf$

Solve: $E = hf\,(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(91.7 \times 10^6\ \text{Hz}) = \mathbf{6.08 \times 10^{-26}\ J}$

b. This energy in joules can be converted into electron volts by dividing by 1.6×10^{-19} J/eV.

$$E = \frac{6.08 \times 10^{-26}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = \mathbf{3.80 \times 10^{-7}\ eV}$$

Example 2: Bart uses a helium-neon laser to align his telescope. The laser emits red light with a wavelength of 633 nm. How much energy, in electron volts, is given off by each photon of laser light?

Solution: First, convert nm to m. $633\ \text{nm} = 6.33 \times 10^{-7}$ m.

Given: $h = 6.63 \times 10^{-34}$ J·s
$\lambda = 6.33 \times 10^{-7}$ m
$c = 3.00 \times 10^8$ m/s

Unknown: $E = ?$
Original equation: $E = \frac{hc}{\lambda}$

$$\textit{Solve: } E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{6.33 \times 10^{-7}\ \text{m}} = 3.14 \times 10^{-19}\ \text{J}$$

This can be converted into electron volts by dividing.

$$E = \frac{3.14 \times 10^{-19}\ \text{J}}{1.60 \times 10^{-19}\ \text{J/eV}} = \mathbf{1.96\ eV}$$

Example 3: Compare the de Broglie wavelengths for a proton and an electron, each traveling at 3.00×10^7 m/s.

Given: $m_p = 1.67 \times 10^{-27}$ kg
$m_e = 9.11 \times 10^{-31}$ kg
$v = 3.00 \times 10^7$ m/s
$h = 6.63 \times 10^{-34}$ J·s

Unknown: $\lambda = ?$
Original equation: $\lambda = \dfrac{h}{mv}$

For the proton:

Solve: $\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}\ \text{J·s}}{(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^{7}\ \text{m/s})} = \mathbf{1.32 \times 10^{-14}\ m}$

For the electron:

Solve: $\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}\ \text{J·s}}{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{7}\ \text{m/s})} = \mathbf{2.43 \times 10^{-11}\ m}$

Therefore, the electron's wavelength is 1800 times larger than the proton's.

Practice Exercises

Exercise 1: The sun is a yellow star and emits most of its radiation in the yellow portion of the spectrum. If the sun's radiation peaks at a frequency of 5.20×10^{14} Hz, how much energy is emitted by one photon of this visible yellow light?

Answer: ____________________

Exercise 2: After applying sunscreen, Cherie lies in the summer sun to get a tan. The ultraviolet light responsible for tanning has a wavelength over 310. nm, while the burning rays can range down to 280. nm. Which ultraviolet photons emit more energy, those that tan or those that burn? How much more?

Answer: ____________________

Exercise 3: Gayle cooks a roast in her microwave oven. The klystron tube in the oven emits photons whose energy is 1.20×10^{-3} eV. What are the wavelengths of these photons?

Answer: ____________________

Exercise 4: During the winter Olympic biathalon trials, Eric is shooting his rifle at a target. What is the de Broglie wavelength of a 10.0-g bullet fired from the rifle at 500. m/s?

Answer: ______________________

Exercise 5: An electron microscope is observing detail on a virus down to 5.0×10^{-9} m. How fast must an electron in the microscope be moving to observe detail this size? (Hint: Due to diffraction effects, an electron's wavelength must be about the same size or smaller than the object being observed.)

Answer: ______________________

18-2 The Photoelectric Effect

Because photons of light carry energy, they can cause electrons to be ejected from certain metal surfaces just by being absorbed by the metal and transferring their energy to the electrons. This process is known as the **photoelectric effect**. However, certain conditions must be met in order for photoelectrons to be ejected.

First, the incoming photon must have enough energy to cause the liberation of an electron. The frequency that corresponds to this amount of energy is called the **threshold frequency**. At the threshold frequency, the photon has just enough energy to free the electron from the surface, and there is no excess kinetic energy given to the emitted electron.

The energy required to free the electron is called the **work function**. Any excess energy given to the electron becomes the kinetic energy that puts the electron in motion. Therefore,

photon energy = kinetic energy + work function or $hf = \text{KE} + W$

The photoelectric effect is an interaction of one photon with one electron. The release of electrons from a surface is a function of the energy they receive. Therefore, you are more likely to witness the photoelectric effect by shining dim blue light on a surface than bright red light, because the blue light has a higher energy per photon.

The wavelengths of colors in the visible light spectrum fall approximately in the following ranges.

Violet light	400–440 nm	Yellow light	530–590 nm
Blue light	440–480 nm	Orange light	590–630 nm
Green light	480–530 nm	Red light	630–700 nm

Radiation falling just below 400 nm is called **ultraviolet radiation**, while that falling just above 700 nm is called **infrared radiation**. Generally, the photoelectric effect only occurs with ultraviolet and visible radiation.

Solved Examples

Example 4: When Doug walks through the entrance to the hardware store, a bell in the back of the store rings, triggered by a photocell whose work function is 2.70 eV. a) What is the threshold frequency of the light shining on the photocell? b) What is the wavelength of the light?

a. *Given:* $E = 2.70\text{ eV}$, $h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s}$

Unknown: $f = ?$

Original equation: $E = hf$

Solve: $f = \frac{E}{h} = \frac{(2.70\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})}{6.63 \times 10^{-34}\text{ J}\cdot\text{s}} = \mathbf{6.51 \times 10^{14}\text{ Hz}}$

b. *Given:* $c = 3.00 \times 10^{8}\text{ m/s}$, $f = 6.03 \times 10^{14}\text{ Hz}$

Unknown: $\lambda = ?$

Original equation: $c = \lambda f$

Solve: $\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\text{ m/s}}{6.51 \times 10^{14}\text{ Hz}} = 4.60 \times 10^{-7}\text{ m} = \mathbf{460\text{ nm}}$ This is blue light.

Example 5: What is the kinetic energy of photoelectrons emitted when ultraviolet light of 200. nm shines on a photocell whose work function is 2.50 eV?

Given: $h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s}$, $c = 3.00 \times 10^{8}\text{ m/s}$, $\lambda = 2.00 \times 10^{-7}\text{ m}$, $W = 2.50\text{ eV}$

Unknown: $\text{KE} = ?$

Original equation: $\frac{hc}{\lambda} = \text{KE} + W$

Solve: $\text{KE} = \frac{hc}{\lambda} - W$

$$= \frac{(6.63 \times 10^{-34}\text{ J}\cdot\text{s})(3.00 \times 10^{8}\text{ m/s})}{2.00 \times 10^{-7}\text{ m}} - (2.50\text{ eV})(1.6 \times 10^{-19}\text{ J/eV})$$

$$= \mathbf{5.95 \times 10^{-19}}\text{ J}$$

Practice Exercises

Exercise 6: The work function for three surfaces are as follows: mercury = 4.50 eV, magnesium = 3.68 eV, and lithium = 2.30 eV. a) At what threshold frequency are electrons liberated from each of these surfaces? b) What color light corresponds to these threshold frequencies?

Answer: **a.** ________________

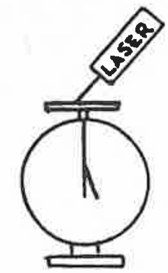
Answer: **b.** ________________

Exercise 7: Shelley shines her red, helium–neon laser, whose wavelength is 633 nm, on a photocell that has a work function of 2.38 eV. a) Will the photocell function with this wavelength of light? b) If so, what is the kinetic energy of the photoelectrons released? If not, what wavelength corresponds to the threshold frequency?

Answer: **a.** ________________

Answer: **b.** ________________

Exercise 8: A classic physics demonstration involves placing a shiny zinc plate on a negatively charged electroscope and shining ultraviolet light on the plate. If the work function of zinc is 4.31 eV and the wavelength of the light is 250 nm, with what kinetic energy are photoelectrons ejected from the zinc plate? b) What will happen to the leaves of the electroscope?



Answer: **a.** ________________

Answer: **b.** ________________

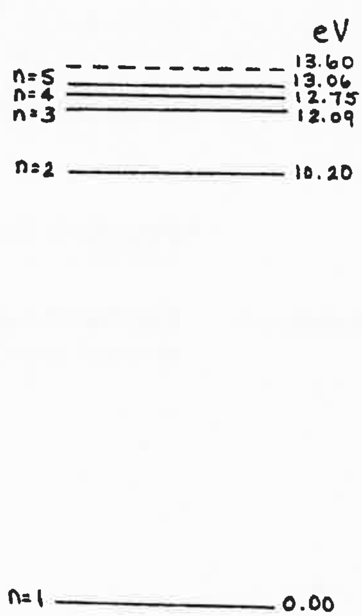
18-3 Energy Level Diagrams

Each atom has its own characteristic set of "fingerprints" or allowed energy states that its electrons can occupy. An **energy level diagram** is a representation of these allowed energy states. The electrons in an atom cannot occupy any level between these allowed states, but instead "jump" from level to level. This is analogous to a person trying to stand *between*, rather than *on*, the rungs of a ladder. It is impossible to do so!

Generally, electrons are found at the lowest energy level or **ground state**. However, when an electron absorbs a photon from its surroundings, it becomes excited and jumps up to a higher energy level. Since the photon is removed from the incident light, this produces an **absorption spectrum**.

When the electron returns to a lower energy level, it emits one or more photons in the process, producing a bright line or **emission spectrum**. If these emitted photons fall in the visible portion of the spectrum, the characteristic spectral lines of the material are seen.

The energy level diagram for hydrogen is shown. Because hydrogen is the most abundant gas in the universe, the hydrogen spectrum has been studied very closely and names have been given to the transitions between energy levels.



Lyman series: Electrons jump to or from the $n = 1$ level. The electromagnetic radiation emitted or absorbed is characterized as ultraviolet.

Balmer series: Electrons jump to or from the $n = 2$ level. The electromagnetic radiation emitted or absorbed is characterized as visible light.

Paschen series: Electrons jump to or from the $n = 3$ level.

Brackett series: Electrons jump to or from the $n = 4$ level.

Pfund series: Electrons jump to or from the $n = 5$ level. The electromagnetic radiation emitted or absorbed from these three series is characterized as infrared.

Solved Examples

Example 6: **a.** What wavelengths of light are emitted by an electron jumping from $n = 2$ to $n = 1$, and from $n = 4$ to $n = 3$? b) To what portion of the electromagnetic spectrum do these wavelengths correspond?

From $n = 2$ to $n = 1$

Given: $E_2 = 10.2$ eV
$E_1 = 0$ eV
$h = 6.63 \times 10^{-34}$ J·s
$c = 3.00 \times 10^8$ m/s

Unknown: $\lambda = ?$
Original equation: $\frac{hc}{\lambda} = E_2 - E_1$

Solve: $$\lambda = \frac{hc}{E_2 - E_1} = \frac{(6.63 \times 10^{-34}\ \text{J·s})(3.00 \times 10^8\ \text{m/s})}{(10.2\ \text{eV} - 0\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} = 1.22 \times 10^{-7}\text{m}$$

From $n = 4$ to $n = 3$

Given: $E_4 = 12.75$ eV
$E_3 = 12.09$ eV
$h = 6.63 \times 10^{-34}$ J·s
$c = 3.00 \times 10^8$ m/s

Unknown: $\lambda = ?$
Original equation: $\frac{hc}{\lambda} = E_4 - E_3$

Solve: $$\lambda = \frac{hc}{E_4 - E_3} = \frac{(6.63 \times 10^{-34}\ \text{J·s})(3.00 \times 10^8\ \text{m/s})}{(12.75\ \text{eV} - 12.09\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} = \mathbf{1.88 \times 10^{-6}\ m}$$

b. 122 nm is ultraviolet and 1880 nm is infrared.

Practice Exercises

Exercise 9: Use the energy level diagram for hydrogen on page 239 to determine the shortest wavelength in the Paschen series of hydrogen.

Answer: ____________________

Exercise 10: The sun's spectrum is made up of many absorption lines called *Fraunhofer lines.* How many electron volts of energy are absorbed in order to produce the H α line whose wavelength is 657.7 nm?

Answer: ______________________

Exercise 11: A stellar spectrum shows three absorption lines of hydrogen produced as electrons move from the $n = 2$ state to higher energy levels ($n = 3$, $n = 4$, $n = 5$). What are the wavelengths and colors of the three lines missing from the continuous spectrum?

Answer: ______________________

Exercise 12: On June 24, 1999, NASA launched FUSE (the Far Ultraviolet Spectroscopic Explorer) to explore the universe using high-resolution spectroscopy in the far ultraviolet spectral region. If FUSE records radiation of wavelength 102.8 nm, between what two energy levels must the electron jump in the hydrogen atom to produce this line?

Answer: ______________________

18-4 Radioactivity

Many atoms that contain large numbers of neutrons are unstable, or **radioactive**. This means that in a given amount of time, the atoms of the substance will decay or change into different atoms with the emission of α, β, or γ rays from the nucleus.

Vocabulary **Activity:** The rate at which a radioactive sample decays.

If a radioactive sample containing N atoms is allowed to decay for an elapsed time, Δt, there will be a change in the number of atoms, ΔN, which depends upon the **decay constant,** λ, for that particular material. Note: This is not the same λ used to represent wavelength. The decay constant is the probability per unit time that a nucleus will decay. The term $\Delta N/\Delta t$ is called the **activity.**

$$\frac{\textbf{change in number of atoms}}{\textbf{elapsed time}} = -(\textbf{decay constant})(\textbf{original number of atoms})$$

or $$\frac{\Delta N}{\Delta t} = -\lambda N$$

The SI unit for activity is the **becquerel (Bq)**, which equals one **decay per second.**

The number of radioactive atoms, N, remaining after a time, t, can be found if you know the number of atoms in the original sample, N_o, and the decay constant of the material, λ.

$$N = N_o\, e^{-\lambda t}$$

where e is the base of the natural logarithm and is approximately equal to 2.72.

Another way of examining radioactivity is by looking at the half-life of a sample.

Vocabulary **Half-life:** The time it takes for half of a radioactive sample to decay.

$$\textbf{Half-life} = \frac{\textbf{0.693}}{\textbf{decay constant}} \quad \text{or} \quad T_{1/2} = \frac{0.693}{\lambda}$$

Solved Examples

Example 7: Cobalt-60, used in radiation therapy for cancer patients, has a half-life of 5.26 y. A sample of cobalt-60 containing 5.00×10^{12} radioactive atoms sits in a lead case in the medical stockroom of St. Mary's Hospital for 10.0 years. How many cobalt-60 atoms remain after this amount of time?

Solution: First, find the decay constant for cobalt-60.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5.26\ \text{y}} = 0.132\ \text{y}^{-1}$$

Given: $\lambda = 0.132\ \text{y}^{-1}$
$N_o = 5.00 \times 10^{12}$ atoms
$e = 2.72$
$t = 10.0$ y

Unknown: $N = ?$
Original equation: $N = N_o e^{-\lambda t}$

Solve: $N = N_o e^{-\lambda t} = (5.00 \times 10^{12}\ \text{atoms})(2.72)^{-(0.131\ \text{y}^{-1})(10.0\ \text{y})}$
$= \mathbf{1.33 \times 10^{12}}$ **atoms**

Example 8: Radioactive gold-198 is used as a tracer to test liver functions in low-level liver scans. Dr. Rogers uses gold-198 in a liver scan on Otis, who has been exhibiting signs of jaundice. A solution containing 3.00×10^9 gold-198 atoms is injected into his liver and observed after 72.0 h. What is the activity of the gold-198 after this amount of time? (Half-life of gold-198 = 2.70 d)

Solution: First, convert days into hours. 2.70 d = 64.8 h

Next, find the decay constant of gold-198.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{64.8\ s} = 0.0107\ \text{h}^{-1}$$

Then, find the number of gold-198 atoms remaining after 72.0 h.

Given: $N_o = 3.00 \times 10^9$ atoms
$e = 2.72$
$\lambda = 0.0107\ \text{h}^{-1}$
$t = 72.0$ h

Unknown: $N = ?$
Original equation: $N = N_o e^{-\lambda t}$

Solve: $N = N_o e^{-\lambda t} = (3.00 \times 10^9\ \text{atoms})(2.72)^{-(0.0107\ \text{h}^{-1})(72.0\ \text{h})} = 1.39 \times 10^9$ atoms

Finally, calculate the activity after converting h into s. 64.8 h = 233 280 s

$$\lambda = \frac{0.693}{233\ 280\ \text{s}} = 2.97 \times 10^{-6}\ \text{s}^{-1}$$

Given: $\lambda = 2.97 \times 10^{-6}\ \text{s}^{-1}$
$N = 1.39 \times 10^9$ atoms

Unknown: $\Delta N/\Delta t = ?$
Original equation: $\Delta N/\Delta t = -\lambda N$

Solve: $\Delta N/\Delta t = -\lambda N = -(2.97 \times 10^{-6}\ \text{s}^{-1})(1.39 \times 10^9\ \text{atoms}) = \mathbf{-4130\ Bq}$

Practice Exercises

Exercise 13: Spent fuel rods contain strontium-90 whose half-life is 28.1 y. Josh works at a nuclear reactor and must safely store the spent rods. If a spent fuel rod contains 1.00×10^{27} atoms of strontium-90 when stored in a sealed container, how many strontium-90 atoms will remain if the container is excavated by archeologists 1000. y later?

Answer: ____________________

Exercise 14: The synthetically manufactured radiopharmaceutical technicium-99 is used to produce a scan of Dale's brain after he suffers a concussion. The half-life of technicium-99 is 6.02 h. What percent of technicium-99 remains in Dale's body 24 h after the scan?

Answer: ____________________

Exercise 15: In the movie *The Planet of the Apes*, the forbidden zone was an area presumably contaminated by the radioactive plutonium fallout from the detonation of nuclear weapons. If Zera finds a rock in the forbidden zone that is tainted with plutonium-239 whose activity is 100. Bq, how many atoms of plutonium does the rock contain when it is discovered? (Half-life of plutonium-239 is 24 900. y)

Answer: ____________________

Exercise 16: In Exercise 15, if the explosion occurred 500. y prior to Zera's discovery, how many plutonium-239 atoms did the rock originally contain?

Answer: ____________________

Additional Exercises

A-1: Gamma rays emitted during the explosion of a nuclear bomb have an energy of 1.2×10^6 eV per photon. What is the frequency of this gamma ray emission?

A-2: An X-ray technician always steps out of the room when the X-ray machine is on. How much energy is carried by each photon of X-ray radiation, if the wavelength of this radiation is 0.0800 nm?

A-3: Mitch is undergoing eye surgery to repair a detached retina. His doctor uses a green laser whose wavelength is 514 nm. How much energy is produced by each laser photon?

A-4: Roy is making holograms with his helium-neon laser. In a helium-neon laser, excited helium atoms collide with neon atoms, raising the neon to an excited state where its energy is 20.66 eV. Stimulated emission then causes electrons in the neon to drop to a lower energy level where $E = 18.7$ eV. What is the wavelength and color of the light given off by a helium-neon laser?

A-5: At Bell Labs in 1926, Davisson and Germer aimed a beam of electrons at a nickel crystal whose atomic spacing was 0.215 nm. If the electrons had a speed of 4.4×10^6 m/s, calculate the de Broglie wavelength of the electrons to determine whether they would be able to pass through the crystal structure or would reflect back.

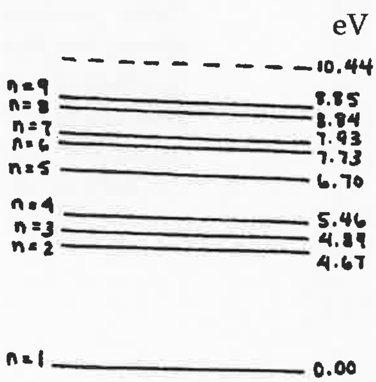
A-6: To determine the size of an oxygen nucleus, protons with kinetic energy of 0.100 GeV (1.00×10^8 eV) are shot at oxygen atoms. a) How fast are the protons moving? b) What is the de Broglie wavelength of the proton?

A-7: Three surfaces, sodium, iron, and gold, have respective work functions of 2.46 eV, 3.90 eV, and 4.82 eV. If light, whose wavelength is 300. nm, shines on each of these materials, which ones will show the photoelectric effect, and what will be the kinetic energy of any photoelectrons emitted?

A-8: Use the energy-level diagram for mercury to determine how much energy is needed to ionize a mercury atom in the $n = 4$ level.

A-9: A mercury atom absorbs a photon of wavelength 161 nm. What energy level does it jump to?

A-10: Hal looks at a mercury vapor street lamp through a diffraction grating and measures the wavelength of a spectral line to be 577 nm. Between what two energy levels must the electron jump to produce this line?



Energy levels for Mercury

A-11: Gloria is testing her basement for radon with a kit she bought at the drugstore. The half-life of radon is 3.83 days and Gloria is informed that 5.00×10^6 radon atoms were present in her basement at the time of testing. Gloria hires a mason to seal off her basement and she runs the test again 30.0 days later. How many radon atoms will now be found in the basement?

A-12: While checking the radioactive tritium levels in the missiles at a Titan missile site, Hugh discovers that it has been 15.0 years since one of the missiles was last inspected. What percent of the radioactive tritium has been depleted? (Half-life of tritium = 12.4 y)

A-13: Nina has a watch whose hands glow in the dark due to a special paint containing radium-226 whose half-life is 1.60×10^3 y. When Nina takes the watch in for a cleaning after 20.0 y, the radium in the hands is found to have an activity of 1.12×10^{14} Bq. How many radium-226 atoms does the watch contain at this time?

A-14: In A-13, how many radium-226 atoms were originally in the watch when it was first purchased 20. years ago?

Challenge Exercises for Further Study

B-1: In Exercise 5, what minimum accelerating voltage of the electron microscope will produce an electron with this de Broglie wavelength?

B-2: Carbon-14 is commonly used to determine the age of organic material. Darlene is on an archeological dig in Mexico and discovers among some ruins what she thinks is an ancient Mayan bone. a) If the bone shows activity of 2.59×10^6 Bq, while the same mass of new human bone shows an average activity of 3.11×10^6 Bq, how old are the excavated bones? (Hint: Half-life of C-14 = 5730 y.) b) Why is carbon-14 a good substance to use for radioactive dating?

B-3: Two containers of radioactive iodine sit on a shelf in Doctor Bailin's supply closet, but the print on the labels has faded and is difficult to read. Dr. Bailin needs some iodine for a thyroid scan but she must only use iodine-131 whose half-life is 8.27 h and not iodione-59 whose half life is 44.6 d. She tests a sample and finds its activity to be 5.00×10^5 Bq. What should the activity be 24.0 h later if Dr. Bailin is testing the iodine-131?